STUDIES ON THE PASSAGE OF HEAVY CHARGED PARTICLES THROUGH MATTER

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By
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STATEMENT

I hereby declare that the work presented in this thesis is the result of investigations carried out by me in the Department of Chemistry, Indian Institute of Technology, Kanpur, India under the supervision of Professor Shankar Mukherji.

In keeping with the general practice of reporting scientific observations, due acknowledgement has been made wherever the work described is based on the findings of other investigators.

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CERTIFICATE I

This is to certify that Mr. Brijesh Kumar Srivastava has satisfactorily completed all the courses required for the Ph.D. degree programme. These courses include:

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Chm 523	Chemical Thermodynamics
Chm 524	Modern Physical Methods
Chm 541	Advanced Inorganic Chemistry I
Chm 542	Advanced Inorganic Chemistry II
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CERTIFICATE II

Certified that the work presented in this thesis entitled: "STUDIES ON THE PASSAGE OF HEAVY CHARGED PARTICLES THROUGH MATTER" has been carried out by Mr. Brijesh Kumar Srivastava under my supervision and the same has not been submitted elsewhere for a degree.

Shankar Mukherji

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The thesis entitled, 'Studies on the Passage of Heavy Charged Particles Through Matter', deals with the formulation of range-velocity and stopping power equations for fission fragments, partially stripped and completely stripped heavy ions in solid as well as in compound media and the calculation of the effective charge (mean equilibrium charge) of a moving ion.

Chapter I introduces ion penetration phenomena and describes broadly the need of the present investigation.

Chapter II describes briefly the range and energy loss concepts and some experimental methods for the determination of ranges and energy-losses.

In Chapter III various theories, such as the classical theory of Bohr, Bethe's quantum mechanical approach, Bloch's formulation, and the semi-quantum mechanical treatment of Bohr, for the study of interaction of charged particles with matter are described.

Chapter IV describes two universal range-velocity and energy-loss equations for fission fragments and partially stripped heavy ions in solid and compound media, obtained through the present investigation. It is found that in the case of light media ($Z<45.5$), the number of orbital electrons

$n(U_s)$ having velocity less than a given velocity U_s is given by,

$$n(U_s) = 0.28\ Z^{2/3}\frac{U_s}{V_o}\ ,$$

where Z is the atomic number of the medium and $V_o = e^2/\hbar$.

Chapter V deals with the development of the stopping power equations for completely stripped heavy ions. These equations have been obtained by considering carefully the limits of integration of the logarithmic terms in Bohr's energy loss equation.

In Chapter VI, calculated values of the effective charges of light ions ($Z \leq 45.5$) in gaseous media are presented and are compared with the corresponding experimental values.

CONTENTS

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CHAPTER I

INTRODUCTION

The passage of energetic heavy charged particles through matter still continues to be of great interest even after sixty years of active investigations.[1] This interest arises out of several theoretical and practical considerations. In 1913, Niels Bohr[2] laid down the theoretical principles for solving this problem on the basis of coulombic interactions between the penetrating charged particle and the orbital electrons of the penetrated medium. Bohr's treatment involves the notion of impact parameters and his great contribution lay in postulating a definite "cut off" point for the impact parameter and in providing a simple, but approximate value for the maximum impact parameter. In 1948, Bohr[3] reconsidered the whole problem and went to great lengths in examining the limits within which the classical orbital picture and the idea of impact parameters may be assumed to be valid. Meanwhile Bethe[4] had

developed his quantum mechanical treatment for calculating the energy loss of charged particles in matter and Bohr[3] proposed a generalized stopping power formula which incorporates the features of his earlier classical formula[2] as well as the features of Bethe's quantum mechanical formula.[4] However, when one attempts to use either Bethe's formula[4] or Bohr's formula,[3] one comes across many practical difficulties.

Firstly, there are definite domains determined by the nature of the penetrating particle and the penetrated media (mass number and nuclear charge) and the velocity of the particle, within which these formulae are valid. Apart from these restricted domains, there are broad regions where none of the above treatments are applicable. For practical purposes one often needs compute the energy-loss of particles which belong to these "no man's land". Further, even within their separate restricted regions of validity, each formula cannot be directly used for stopping power calculation because each contains some parameters whose values are not obtainable through rigorous theoretical calculations. Thus, inspite of the existence of Bohr's earlier formula[2] for sixty years and Bethe's formula[4] for forty years, most of the practically used stopping power formulae suffer from considerable amount of empiricism.[1] Secondly, rapid development has taken place

in the generation of medium energy charged particle accelerators in the last decade. This development has revived interest in the stopping powers for the heavy ions produced by these accelerators. In the current literature terminology, "heavy" ions mean particles much heavier than protons. These heavy ions, unless they are accelerated to a very high energy, remain partially stripped of their orbital electrons, and while passing through matter gradually pick-up electrons and become progressively neutralized as they slow down. This phenomenon introduces a further complicating factor in the stopping power formulae mentioned earlier, since these were based on the assumption that the ionic charge state of the ion remains invariant throughout. A majority of these heavy ions belong to the domain in which neither Bohr's[2] classical treatment nor Bethe's[4] quantum-mechanical formulation is valid. Fission fragments belong to this class of heavy ions and because of their practical importance in a variety of fields such as fission-chemistry[5] and direct energy conversion in fission-electric cells,[6] the total penetration depths or ranges of fission fragments as well as their rate of energy loss in different materials, have been the subject of numerous investigations.[7-10]

Recently, Lindhard et al.[11] put forward a stopping power formula which was assumed to hold for heavy ions and

fission fragments. However, disagreement exists between the experimental data and the corresponding theoretically calculated values.[8] Practical range-velocity relationship remained completely empirical till Niday[12] proposed his range-velocity equation. This equation was obtained by modifying Bohr's generalized stopping power formula[3] and was found to be fairly precise in the case of a large number of fission-fragments penetrating metallic uranium. The purpose of the present investigation is (a) to obtain very general stopping power and range-velocity equations for partially stripped heavy ions and (b) to derive stopping power equations for completely stripped ions which belong to neither of the domains in which Bohr's or Bethe's equation is valid.

CHAPTER II

EXPERIMENTAL METHODS FOR DETERMINING THE VELOCITY, RANGE, AND ENERGY-LOSS OF FISSION FRAGMENTS AND CHARGED HEAVY PARTICLES

II.1 FISSION-FRAGMENT VELOCITY

The time-of-flight technique, originated by Leachman[13] consists of finding out the time required by a fragment to travel a fixed distance from its point of origin and thus its velocity can be obtained. The time of origin is provided by the pulse from the electrons emitted from the backing of the fissile source travelling the 1 cm. distance from the fission source to the nearest scintillation detector. The time-of-flight of the complementary fragment through the 343 cm drift distance determined the time of occurrence of the pulse from the remote detector. Utilizing this technique Leachman[13] measured the velocity distribution of fragments from the fission of ^{233}U, ^{235}U and ^{239}Pu. Comparison of the results of the velocity distribution from the time-of-flight technique and ionization measurement shows that the latter

method gives lower values for the kinetic energy due to an inherent 'ionization defect'.

Stein[14] used two evacuated drift tubes with a thin fissile material placed between them, at the end of which two scintillation detectors are placed. The zero time is recorded from a pulse generated by a large number of electrons (≈50-100) stripped out of the thin backing foil, when one of the fragments passes through it on its way down the drift tube. These electrons are accelerated to a high potential and focussed on the photomultiplier tube, the signal produced thus gives the beginning of the fission event with a time resolution of 5×10^{-9} sec.

II.2 RANGE-CONCEPT AND METHODS OF RANGE-DETERMINATION

The total distance travelled by a charged particle in an absorbing medium before it comes to rest is equal to its range. Assuming a uniform linear motion, the greater the kinetic energy, the more would be the penetration depth. A number of techniques have been used for range determination of the fission products.

II.2.1 Fission Products' Ranges

(a) Thin target method:

A target can be thin or thick as compared to the range of the particular recoiling product in the target. For example,

for a range of 20 $\mu g/cm^2$ of the particular recoil a target of 1 $\mu g/cm^2$ is a 'thin target'. If the source is very thin and the collimator is so narrow that the recoil products travelling only at right angles to the catcher are allowed to penetrate the catcher then, theoretically, all the products of identical mass should come to a stop at a particular depth of the catcher, which is equal to the range of the product in that catcher material. However, this does not happen in experiment because of the combination of two factors: (a) on account of the statistical nature of the collision between the products and the absorber material, which determines the energy loss of former, some products come to stop a little before and some a little beyond the thickness at which they should stop and (b) there is an initial spread in the kinetic energies of the products of identical mass. The ranges with thin targets are determined by two methods.

(1) Thin target-thin catcher differential method: In this method, a well collimated beam of fission products, emerging from a thin layer of the fissile material is allowed to penetrate the absorber which is either in the form of stack of thin foils[15] or a thick foil from which thin layers[16] are removed later either by chemical stripping or by fine grinding. The activity in each thin foil or each layer removed is measured by radiochemical techniques.

From the plot of the activity in each layer vs the thickness of that layer one may obtain the mean or most probable range of a particular fission product. This method requires sufficient activity in each thin layer so that counting errors may not be too high. The possible sources of error in this method lies in the finite thickness of the fissile material, the finite size of the collimating holes, and in the non-uniformity in the thickness of the thin foils or the thin layers removed.

(2) Thin target-thin catcher integral method: In this method a thin target and several thin catcher foils are stacked together and irradiated for a long time without using a collimater. The activity 'A_t' present beyond a thickness 't' from the surface of the stack facing the target is related to the range 'R' as given by Segre' and Wiegand,[17]

$$A_t = C\,(R-t),$$

where C is a constant. Thus, a plot of A_t vs t gives a straight line the extrapolation of which to the 't' axis gives the range R.

(b) Thick Target Method:

Thin target methods become unsuitable for low fission-cross section materials, particularly when fission is induced

by accelerated charged particles or fast neutrons. The main disadvantage with accelerated charged particles is that the beam intensity decreases with increasing energy of the particle and to have a measurable activity, the thin target-thin catcher method would require long periods of irradiation. Sugarman et al.[18] developed the thick-target integral and differential methods for the range measurements. The integral method gives the range value in the target material while the range in catcher material is obtained by the differential method.

(1) Thick target-thick catcher method: Niday[12] used this method for measuring ranges of nuclides from the thermal neutron induced fission of ^{235}U. A thick foil of the target material is sandwiched between two absorber foils (thick enough to stop all the recoiling nuclides) and irradiated with thermal neutrons. If it is assumed that the fission fragments are emitted isotropically, then the range R of the nuclide in the target material is given by

$$R = \frac{A_C}{A_C + A_t} \cdot 4.t$$

where A_C is the activity of the nuclide in one of the catcher foils, A_t is the activity of the nuclide in the target and 't' is the thickness of the target foil. Chinaglia et al.[19] have used modified form of this method to determine the

ranges in carbon using high resolution Ge(Li) detector. With this technique it has been possible to measure the ranges of several fission products without doing any chemical separation. A homogeneous source was obtained through thermal diffusion of enriched uranium at a high temperature into pyrolytic carbon discs. But this technique is limited to nuclides that have reasonable yields and emit prominent gamma rays. Gordon, et al.[20] were the first to develop this technique. The accuracy of range measurements obtained with Ge(Li) spectrometry is potentially at least as high as for the standard radiochemical methods, and the former can be used for volatile products that are difficult to handle radiochemically.

(2) Thick target-thin catcher method: Alexander and Gazdik[21] used moderately thick targets sandwiched between stacks of absorber foils of different thicknesses. The average range R of a product may be calculated from the equation:

$$F_t = \frac{1}{2}\left(1 - \frac{t}{\bar{R}} - \frac{C w}{2\bar{R}}\right)$$

where F_t is the activity of the recoils penetrating beyond a thickness 't' of the catcher assembly, $\bar{R}$ is reciprocal of the average range, w is the target thickness and C is a constant for converting the rate of velocity loss in the target material to that in the catcher material and is evaluated from

the expression:

$$\frac{\Delta F_t}{\Delta w} = -\frac{C}{4\bar{R}}$$

II.2.2 Ranges of Accelerated Heavy Ions

The differential range of accelerated heavy ions is obtained by performing integration of the energy-loss data:

$$R(E) - R(E_o) = \int_{E_o}^{E} \left(\frac{dE}{dx}\right)^{-1} dE$$

where $R(E)$ and $R(E_o)$ are the ranges corresponding to energies E and E_o respectively. Using this procedure Bridwell and Moak[22] determined the ranges of ^{79}Br and ^{127}I ions in Be, Al, Ni, Au and UF_4.

II.3 ENERGY-LOSS MEASUREMENT

Experimental measurement of the stopping powers of heavy ions requires accurate knowledge of the energies of the ion before and after it has passed through a thin absorber foil of precisely known thickness. The ratio of the energy loss to the foil thickness is regarded as the stopping power at a mean energy defined by

$$E_m = E_i - \frac{\Delta E}{2}$$

where E_i is the incident beam energy, ΔE is the energy loss in foil and $\Delta E \ll E_i$.

II.3.1 Fission Fragments

The experimental set-up for the measurement of energy-loss of fission-fragments consists of absorbers of different thicknesses mounted on rings and positioned approximately on the mid-plane between the fissile source and the solid state detector in an evacuated chamber. If only one detector is used, then the energy-loss of the average fission fragment can be obtained.[23] Using two detectors on both the sides of the fissioning material (thin-source), the energy loss of the median light and the median heavy fragments may be obtained by appropriate coincidence circuits.[9,24] But none of these measurements give information regarding the energy loss of each of the observed fission fragments (variation of stopping-power with mass). The experiment for the measurement of the energy loss of individual fission fragments involves the evaluation of three parameters[10]: the energy of the primary fission fragments, the time necessary for it to traverse a measured distance and the energies of the complementry fragments.

The fission fragments after passing through the absorber, impinge on the detector and the recorded pulse-height spectrum yields the fragment energy as follows:

Schmitt et al.[25] showed that the energy E of the fragment is related to the pulse-height and the mass A of the fragment by the expression:

$$E = (a + a'A)P + b + b'A$$

where a, a', b and b' are detector constant and given by

$$a = 24.0202/(P_L - P_H)$$

$$a' = 0.03574/(P_L - P_H)$$

$$b = 89.6083 - a\ P_L$$

$$b' = 0.1370 - a'\ P_L$$

P_L and P_H are the peak positions of the light and the heavy fission fragments, and P is the pulse height as determined from a fission fragment kinetic energy spectrum.

II.3.2 Charged Particles

A beam of accelerated heavy ions is passed through an analyzing magnet of high resolution to ensure that the initial energy is constant and well defined. This mono-energetic beam is then allowed to travel a known thickness of the absorber foil placed in its path. The energy of the transmitted ions is then measured using either an electrostatic analyzer equipped with a position sensitive detector,[26] a magnetic spectrograph,[27] a solid state detector[28] or a time-of-flight equipment.[26]

The measurement of the absorber foil thicknesses is done either by weighing each foil on a semi-microbalance and

measuring the diameter of foils with a comparator or by the α-particle energy-loss method. In the latter procedure the thickness of each absorber foil is determined by measuring the degraded energy of the ^{252}Cf or ^{244}Cm α-particles. Use is made of the α-particle range and stopping power data.

CHAPTER III

THEORETICAL ASPECTS OF STOPPING PROCESS

III.1 ENERGY-LOSS PROCESS

The mechanism by which a swiftly moving charged particle loses its kinetic energy involves four types of interactions.

III.1.1 Inelastic collision with the atomic electrons

Inelastic collisions involve excitation and ionization of the atoms of the medium. The energy imparted by the moving particle to the atoms of the medium causes ionization. Presumably, both ionization and excitation occur simultaneously. The slowing down of the moving particle due to this phenomenon is also known as "electronic stopping".

III.1.2 Elastic collision with the nucleus

This type of interaction , also described as nuclear collision, is equivalent to a two-body billiard ball like collision between the moving particle and the heavy nucleus

of the stopping medium. This process involves the transfer of kinetic energy to the stopping atom as a whole.

III.1.3 Inelastic collision with a nucleus

A close non-capture encounter of the moving particle with the nucleus of the stopping atoms causes a deflection of the moving particle and frequently leads to the emission of a quantum of radiation appearing in the form of bremsstrahlung. The intensity of the radiation is inversely proportional to the square of the mass of the particle and is not of much importance in the case of heavy ions.

III.1.4 Elastic collision with the atomic electrons

In this type of interaction the elastic deflection of the moving particle in the field of the atomic electrons of the stopping atoms takes place. The energy transfer is generally less than the lowest excitation potential of the electrons. Such collisions are significant only in the case of very low energy electrons.

Electronic collision is usually the predominant mechanism by which a charged particle loses its energy. Nuclear collision starts dominating when the ion velocity becomes less than the velocity of the electron in the first Bohr orbit. In the following section the theory of electronic collisions will be presented.

III.2 ELECTRONIC STOPPING PROCESS

A detailed discussion of the interaction between a moving charged particle and an electron of the stopping medium was put forward by Bohr[2] in 1913. Using classical mechanical ideas an expression for the rate of energy loss was obtained. Later on in 1930 Bethe[4] derived an expression for the energy loss of high speed light particles. Three years after Bethe's quantum mechanical treatment, Bloch[29] succeeded in getting a stopping power equation which incorporates the classical treatment of Bohr[2] and the quantum-mechanical derivation of Bethe[4] . In 1948 Bohr[3] again considered the phenomenon of charged particle energy-loss and derived a stopping power equation which incorporates both classical as well as quantum mechanical ideas.

III.2.1 Classical theory of Bohr

The interaction between a charged particle and an atomic electron is described as a coulomb interaction between the electron and a positive point charge. Thus, if a heavy ion decreases its charge by picking up electrons in its passage through matter the rate of energy-loss will diminish. The energy transfer to an electron in a collision with an ion of velocity V is given by

$$\Delta E = \frac{2z^2e^4}{m\,p^2V^2}\,, \qquad \ldots(3.1)$$

where m and e are the electronic mass and charge, p is defined as the closest distance of approach of the moving ion to the electron and is termed as 'impact parameter' and ze is the charge of the moving ion. In the derivation of Eq.(3.1) it is assumed that

(a) the electron is free and initially at rest, because in each collision the struck electron receives so much energy that its initial binding energy can be neglected. This is justified as long as the velocity of the ion is greater than that of the electron.

(b) The ion moves almost undeflected, because the mass of the electron is negligible compared to that of the heavy ion.

In Eq.(3.1), ΔE is the energy gained by one electron. The energy gained by the electrons lying between impact parameters p and p+dp and in a small thickness dx of the stopping medium is given by

$$dE = \Delta E \times \text{number of electrons} = \Delta E \cdot 2\pi p\, n\, dp\, dx$$

$$dE = \frac{4\pi z^2 e^4 n}{m\, p\, v^2}\, dx\, dp \qquad \ldots (3.2)$$

where n is the number of electrons per unit volume. The total energy loss per unit path length is given by

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4 n}{m\, v^2} \int \frac{dp}{p}\,, \qquad \ldots (3.3)$$

The limits of integration are given by the values of p_{max} and p_{min}. The maximum value for the impact parameter p_{max} is given by the time of interaction τ of the ion with the electrons, which should be shorter than the natural period of vibration of the electron. The upper limit for the impact parameter is then

$$p_{max} = \tau v = \frac{1.123\ v}{\omega} , \qquad \ldots(3.4)$$

where ω is the cyclic frequency of the electron in the atom of the medium. The minimum impact parameter is determined by the maximum amount of energy that can be imparted to an electron. From Eq.(3.1)

$$p_{min} = \frac{z\ e^2}{m\ v^2} \qquad \ldots(3.5)$$

Using Eqs.(3.4) and (3.5) in Eq. (3.3), the total energy loss becomes

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4 n}{m\ v^2} Z_2 \ln \frac{1.123\ m\ v^3}{z\ e^2\ \omega} , \qquad \ldots (3.6)$$

if all the electrons have the same characteristic frequency ω and Z_2 is the atomic number of the medium. If each atom contains Z_2 electrons having individual frequencies $\omega_1, \omega_2, \ldots \omega_s$, the average energy loss becomes

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4 n}{m\ v^2} \sum_{s=1}^{s=Z_2} \ln \frac{1.123\ m\ v^3}{z\ e^2\ \omega_s} \qquad \ldots(3.7)$$

Eq.(3.7) was first derived by Bohr[2] using simple classical mechanical ideas.

III.2.2 Criterion for the applicability of the classical orbital picture

Later in 1948 Bohr[3] showed that the classical theory is valid only for those collisions in which the ion velocity V greatly exceeds the orbital velocity U_s of the atomic electrons in their respective orbits. This imposes the restriction that V should exceed the velocity of the inner shell electrons (K - Shell) for ionization in the K - Shell. The second difficulty arises due to the fact that the maximum impact parameter must exceed the radius of the Bohr orbit ($p_{max} > a_s$). This gives the minimum energy transfer to the electrons, which are less than the ionization potentials and are, therefore, inconsistent with classical ideas regarding the energy transfer to orbital electrons. The impact parameter of a particle approaching a force centre can be precisely determined only when the incident particle is well collimated. This collimation can be achieved only with a narrow aperture but which would, however, cause a diffraction of the particle. By considering the diffraction caused by the aperture and the deflection of the particle, Bohr[3] showed that

$$\chi \gg 1 \quad , \qquad \ldots(3.8)$$

is the essential condition for the validity of the classical orbital picture, where $\chi = 2\,ze^2/(\hbar V)$ and ze is the charge of the moving particle of velocity V. For decreasing values of χ, the orbital picture gradually loses its validity.

III.2.3 Bethe's quantum mechanical approach

Bethe[4] in 1930 developed his theory to account for the energy loss of fast moving light particles for which the classical theory predicted too great an energy-loss. Bethe's[4] treatment is based on the Born approximation, applied to collisions between the heavy particle and the atomic electrons. The Born approximation requires that

$$\frac{z\,e^2}{\hbar V} \ll 1\,, \qquad \ldots(3.9)$$

where ze and V are the charge and velocity of the moving particle. The above condition is well satisfied for high velocity and low-charge of the moving particle. The energy loss per unit path length is given by

$$\frac{dE}{dx} = nZ_2 \int T\, d\sigma, \qquad \ldots(3.10)$$

where $d\sigma$ is the Rutherford differential cross-section, T is the energy lost and n is the number of atoms per unit volume of the medium of atomic number Z_2. Integrating Eq. (3.10) between the limits T_{max} and T_{min},

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mV^2} nZ_2 \ln \frac{T_{max}}{T_{min}}, \qquad \ldots(3.11)$$

where T_{max} is given by the maximum energy transfer by the incident particle to the electron i.e. $2mV^2$, while T_{min} is the energy necessary to excite the electrons of the stopping atoms. These considerations, when applied to Eq.(3.11), gives

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mV^2} nZ_2 \ln \frac{2mV^2}{\bar{I}}, \qquad \ldots(3.12)$$

where $\bar{I}$ is the mean excitation potential of the atoms of the medium. For relativistic velocities of the incident particle the energy loss is given by

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mV^2} nZ_2 \left\{ \ln \frac{2mV^2}{(1-\beta^2)\bar{I}} - \beta^2 \right\}, \qquad \ldots(3.13)$$

where $\beta = V/c$ and c is the velocity of light.

III.2.4 Bloch's relation

Bloch[29] also derived an expression for the electronic energy loss rate, which incorporates the quantum-mechanical equation of Bethe, and the classical formula of Bohr, as limiting cases for small and large values of the parameter $ze^2/(\hbar V)$ This equation has the following form

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mV^2} nZ_2 \left\{ \ln \frac{2mV^2}{\bar{I}} + \Psi(1) - R\,\Psi\left(1+i\frac{ze^2}{\hbar V}\right) \right\}, \qquad \ldots(3.14)$$

where ψ is the logarithmic derivative of the gamma-function and R ψ denotes the real part of ψ. For $ze^2/(\hbar V) \gg 1$. Eq. (3.14) reduces[2] to Eq.(3.7) and for $ze^2/(\hbar V) \ll 1$ it is Bethe's relation i.e. Eq.(3.12). But for intermediate values of $ze^2/(\hbar V)$, Eq. (3.14) is approximately equal to either of the Eqs. (3.7 and (3.12).

Bloch[29] also showed, on the basis of the Thomas-Fermi statistical approach, that the mean excitation potential $\bar{I}$ of an atom should be proportional to the atomic number Z_2

$$I = K Z_2 \qquad \ldots(3.15)$$

The empirically determined values of the "Bloch Constant" K do turn out to be approximately the same for large values of Z_2. But for media of low atomic numbers, values of K are found to be unexpectedly higher than those predicted by Eq. (3.15).

III.2.5 Bohr's semi-quantum mechanical treatment of electronic collisions

Three decades after the development of his classical theory of energy-loss, Bohr[3] turned to a more detailed consideration of this problem. Bohr[3] dealt with electronic collisions in detail by taking care of both excitation and ionization as well as the quantum mechanical "resonance effect".

The classical approach in terms of a definite impact parameter is valid only when $\chi \gg 1$ whereas $\chi \ll 1$ is required

for the Born approximation on which Bethe's theory is based, where χ is given by

$$\chi = 2\, ze^2/(\hbar V), \qquad \ldots(3.16)$$

and V is the velocity of the ion of charge ze. The classical energy loss Eq. (3.7) was put by Bohr in the form

$$\Delta E = n\, \Delta x \cdot B_\epsilon \sum_s \ln\, (k \frac{T_m}{D_s}), \qquad \ldots(3.17)$$

where $k = 1.123$, T_m is the maximum energy transfer by the moving particle to an electron of the medium while D_s is the energy transfer in a free collision with impact parameter $p = d_s$. The other symbols are defined as follows:

$$B_\epsilon = 2\pi z^2 e^4/(mV^2), \qquad \ldots(3.18)$$

$$\frac{T_m}{D_s} = \left\{ \frac{mV^3}{ze^2\omega_s} \right\}^2 = \left\{ \frac{mV^3}{zV_o I_s} \right\}^2, \qquad \ldots(3.19)$$

where the ionization potential $I_s = \hbar\omega_s$ and $V_o = e^2/\hbar$. On the other hand Bethe's quantum mechanical treatment gives

$$\Delta E = 2n\, \Delta x \cdot B_\epsilon \sum_s \ln \frac{T_m}{I_s} \qquad \ldots(3.20)$$

where $T_m = 2mV^2$ and I_s is the excitation potential of the s^{th} electron. For $T_m \gg I_s$, which is assumed in both the

treatments. Eq.(3.19) can be rearranged to give

$$\frac{T_m}{D_s} = \left\{ \frac{2 m V^2}{I_s} \right\}^2 \cdot \left\{ \left(\frac{1}{2z \frac{V_o}{V}} \right) \right\}^2 = \left\{ \frac{T_m}{I_s} \right\}^2 \chi^{-2} \quad , \qquad \ldots(3.21)$$

where $(2 m V^2)$ and $(2z \frac{V_o}{V})$ have been replaced by T_m and χ. Eq.(3.21) shows that, apart from the factor k which may be taken as unity in Eq.(3.17), Eqs.(3.17) and (3.20) become identical for $\chi = 1$, on the assumption that the mean ionization potential and the mean excitation potential are almost identical. For highly charged heavy particles like fission fragments, classical treatment, Eq. (3.17), can be utilized to obtain the energy loss, which is based on the fact that $V \gg U_s$ and $b \ll a_s$, with

$$b = \frac{z e^2}{m V^2} \quad , \qquad \ldots(3.22)$$

$$\text{and} \quad a_s = \frac{\hbar}{m U_s} \quad , \qquad \ldots(3.23)$$

where m and e are the electronic mass and charge, U_s is the orbital velocity of the s^{th} electron with orbital extension a_s and b is the collision diameter. $b \ll a_s$ is always fulfilled for fast moving light particles, but for heavy ions, for some values of V, b may exceed a_s and in which case d_s, the impact parameter $p = d_s$ corresponding to energy transfer D_s, will not give an effective adiabatic limit. In such cases $i_s > d_s$, where i_s is the impact parameter corresponding to

energy transfer I_s and the energy loss may be somewhat larger than given by Eq. (3.17). Two modes of energy transfer may be considered to take into account the collisions for which $a_s \gg b$ and $a_s \ll b$.

(a) Free collisions: If the distance of closest approach is of the order of atomic dimensions, the interaction involves the passing particle and one of the atomic electrons and the energy transfer lies between I_s and the maximum energy transfer T_m. This process is often described as free collisions.

(b) Resonance effects: If the distance of closest approach is large compared to the dimension of the atom, the atom reacts as a whole to the variable field set up by the passing particle. The result is that the probability of excitation or ionization in a single collision is small and the energy transfer T lies between D_s and I_s ($D_s \lesssim T \lesssim I_s$), D_s being the energy transfer corresponding to an impact parameter d_s.

Thus the total energy-loss is due to free collisions and the contribution from the resonance effects. Two cases arise depending on the values of χ, defined by Eq. (3.16). In the following sections (a) and (b) contributions of free collisions and resonance effects will be derived for two values of χ, as shown by Bohr.[3]

(i) $\chi > 1$

For the impact parameter $p = i_s$ the corresponding energy transfer $T = I_s$, for free collisions, is given by

$$I_s = \frac{2z^2e^4}{mV^2} \cdot \frac{1}{i_s^2} , \qquad \ldots(3.24)$$

where ze is the charge of the ion of velocity V. Eq.(3.24) may be rearranged by using $I_s = \frac{1}{2} m U_s^2$ to obtain

$$\frac{1}{2} m U_s^2 = \left(\frac{2ze^2}{V\hbar}\right)^2 \cdot \frac{1}{2m} \frac{\hbar^2}{i_s^2}$$

$$\text{or } i_s^2 = \left(\frac{2ze^2}{V\hbar}\right)^2 \left(\frac{\hbar}{mU_s}\right)^2 \qquad \ldots(3.25)$$

But $\chi = \frac{2ze^2}{\hbar V}$ and $a_s = \frac{\hbar}{mU_s}$, hence Eq.(3.25) reduces to

$$i_s = \chi a_s \qquad \ldots(3.26)$$

The maximum value of the impact parameter p is d_s corresponding to energy transfer D_s in a free collision and is given by the time of interaction τ of the ion with the electron, which should be shorter than the natural period of vibration ω_s of the electron. The condition for this is

$$p = d_s = \frac{V}{\omega_s} = \frac{V\hbar}{I_s}$$

$$\text{or } d_s = V\hbar/\left(\frac{1}{2} m U_s^2\right) = \left(\frac{2V}{U_s}\right)\left(\frac{\hbar}{mU_s}\right)$$

$$d_s = \eta_s a_s , \qquad \ldots(3.27)$$

where $\eta_s = \frac{2V}{U_s}$ and $a_s = \frac{\hbar}{mU_s}$.

From Eqs. (3.26) and (3.27) $\frac{d_s}{i_s}$ is given by

$$\frac{d_s}{i_s} = \eta_s \chi^{-1} \qquad \ldots(3.28)$$

From the above equation for, $\frac{\chi}{\eta_s} < 1$, i_s is always less than d_s and collisions with impact parameter $p < i_s$ may be considered as free and the energy transfer T corresponding to the impact parameter p is always greater than I_s. The energy loss in free collisions is given by

$$(\Delta E)_f = n \Delta x B \epsilon \ln \frac{T_m}{I_s} \qquad \ldots(3.29)$$

From Eq.(3.26) for $\chi > 1$, $i_s > a_s$ and the collisions with $p > i_s$ are of resonance type. In this case the energy transfer T is less than I_s but exceeds D_s, corresponding to the impact parameter d_s as given by Eq.(3.27). The contribution of the resonance effect to the energy loss becomes

$$(\Delta E)_r = n \Delta x B \epsilon \ln \frac{I_s}{D_s} \qquad \ldots(3.30)$$

(T_m / I_s) and (I_s / D_s) in Eqs. (3.29) and (3.30) may be obtained as follows:

$$\frac{T_m}{I_s} = \frac{2mV^2}{\frac{1}{2} mU_s^2} = \left(\frac{2V}{U_s}\right)^2 = \eta_s^2 \qquad \ldots(3.31)$$

$$\frac{I_s}{D_s} = \frac{\frac{1}{2} m U_s^2}{2z^2 e^4/(mV^2)} d_s^2 = \left(\frac{1}{2z \frac{V_o}{V}}\right)^2 \left(\frac{1}{\hbar/mU_s}\right) d_s^2$$

$$= \chi^{-2} a_s^{-1} d_s^2 \qquad \ldots(3.32)$$

Using the value of d_s from Eq. (3.27) in Eq.(3.32), I_s/D_s becomes

$$\frac{I_s}{D_s} = \eta_s^2 \chi^{-2} \qquad \ldots(3.33)$$

Substituting the values of T_m/I_s and I_s/D_s in the expressions for $(\Delta E)_f$ and $(\Delta E)_r$

$$(\Delta E)_f = n \Delta x B\epsilon \ln \eta_s^2 \qquad \ldots(3.34)$$

$$(\Delta E)_r = n \Delta x B\epsilon \ln \eta_s^2 \chi^{-2} \qquad \ldots(3.35)$$

From the above equations it is clear that for $\chi > 1$, $(\Delta E)_f > (\Delta E)_r$. The assumption on which Eq. (3.34) rests is $\frac{\chi}{\eta_s} < 1$ and this leads to the inequality

$$V > \frac{U_s}{2} \chi , \qquad \ldots(3.36)$$

where $\eta_s = \frac{2V}{U_s}$. But there are certain velocity regions where $\left(\frac{\chi}{\eta_s}\right)$ may exceed unity and in this case the energy loss given by Eq. (3.34) will not be valid.

For $\frac{\chi}{\eta_s} > 1$, from Eq. (3.28), i_s exceeds d_s and the values of the impact parameter $p = d_s$ for which collisions

were "free" no longer extend to i_s, since for impact parameters $p > d_s$ the duration of the encounter τ exceeds $\frac{1}{\omega_s}$. Bohr[3] showed that the limiting value d_s^* of the impact parameter for which the probability of ionization is still unity, is given by

$$(d_s^*)^2 = i_s . d_s \qquad \ldots(3.37)$$

Using the values of $i_s = \chi a_s$ and $d_s = \eta_s a_s$, from Eqs. (3.26) and (3.27), in the above equation,

$$(d_s^*)^2 = \chi \eta_s a_s^2 , \qquad \ldots(3.38)$$

If D_s^* is the energy transfer corresponding to the impact parameter d_s^*, then the energy loss in free collisions is given by

$$(\Delta E)_f = n \, \Delta x \, B\epsilon \, \ln \frac{T_m}{D_s^*} . \qquad \ldots(3.39)$$

The value of $(\frac{T_m}{D_s^*})$ may be obtained as follows:

$$\frac{T_m}{D_s^*} = \frac{2mV^2}{2z^2e^4/(mV^2)} . d_s^{*2} = \left\{ \frac{1}{2z \frac{V_o}{V}} \right\}^2 (\frac{1}{\hbar/mU_s})^2 (\frac{2V}{U_s})^2 . d_s^{*2}$$

Putting the values of d_s^* from Eq.(3.38) and $\chi = 2z \frac{V_o}{V}$, $a_s = \frac{\hbar}{mU_s}$ and $\eta_s = \frac{2V}{U_s}$ in the above equation, $\frac{T_m}{D_s^*}$ becomes

$$\frac{T_m}{D_s^*} = \eta_s^2 \left[\frac{\chi}{\eta_s} \right]^{-1} , \qquad \ldots(3.40)$$

and the energy loss is given by

$$(\Delta E)_f = n\Delta x\, B\epsilon \ln \eta_s^2 \left[\frac{\chi}{\eta_s}\right]^{-1}, \qquad \ldots(3.41)$$

Eq. (3.41) holds for $T_m \gg D_s^*$ and $\frac{\chi}{\eta_s} > 1$.

The total energy loss due to free collisions and the resonance effects will be the sum of Eqs.(3.34), (3.35) and (3.41),

$$(\Delta E) = n\Delta x\, B\epsilon \left\{\ln \eta_s^2 [\chi]^{-2} + \ln \eta_s^2 \left[\frac{\chi}{\eta_s}\right]^{-1}\right\}, \qquad \ldots(3.42)$$

where the quantity with in the square bracket is to be replaced by unity if $\frac{\chi}{\eta_s} < 1$.

(ii) $\chi < 1$

For $\chi < 1$, classical mechanical ideas fail completely in accounting for the individual collision effects. But the division between the free collisions and resonance effects is still valid for impact parameters $p > a_s$ and $p < a_s$, where a_s is the orbital extension of the s^{th} electron. The free collision part is still given by Eq. (3.29) while for $p < a_s$, which corresponds to distant collisions (resonance effects), the energy loss is

$$(\Delta E)_r = n\Delta x\, B\epsilon \ln \frac{A_s'}{D_s'}, \qquad \ldots(3.43)$$

where A_s' is the energy transfer corresponding to the impact

parameter $p = a_s$ and is given by

$$A_s' = \frac{2z^2e^4}{mV^2}\,\frac{1}{a_s^2} = \left(\frac{2zV_o}{V}\right)^2 \left(\frac{\hbar}{mU_s}\right)^2 \frac{1}{a_s^2} \cdot \frac{1}{2}\,mU_s^2 \,.$$

$$\text{or} \quad A_s' = \chi^2 . I_s \,, \qquad \ldots(3.44)$$

$$\text{and} \quad \frac{A_s'}{D_s'} = \frac{I_s}{D_s'}\,\chi^2 = \eta_s^2 \,, \qquad \ldots(3.45)$$

where $\frac{I_s}{D_s'}$ is given by Eq. (3.33). Incorporation of Eq.(3.45) in Eq.(3.43) leads to

$$(\Delta E)_r = n\Delta x \;\; B\epsilon \ln \eta_s^2 \qquad \ldots(3.46)$$

$$\text{and} \quad (\Delta E)_f = n\Delta x \;\; B\epsilon \ln \eta_s^2 \qquad \ldots (3.47)$$

Thus for $\chi < 1$, the total energy loss is given by

$$(\Delta E) = 2\, n\Delta x \;\; B\epsilon \ln \eta_s^2 \qquad \ldots(3.48)$$

Eqs. (3.42) and (3.48) combined together give a comprehensive energy loss equation

$$-\frac{\Delta E}{\Delta x} = n\, B\epsilon \left(\sum_s \ln \eta_s^2 [\chi]^{-2} + \sum_s \ln \eta_s^2 \left[\frac{\chi}{\eta_s}\right]^{-1}\right) , \qquad \ldots(3.49)$$

where the terms within the square brackets, if less than unity, should be replaced by unity and the summation of the logarithmic terms include the velocities of all those orbital electrons

which interact with the moving ion at a given ion velocity V. The other symbols are defined as follows:

$$B\epsilon = 2\pi z^2 e^4/(mV^2) \qquad \ldots(3.50)$$

$$\eta_s = 2V/U_s \qquad \ldots(3.51)$$

$$\chi = 2zV_o/V \qquad \ldots(3.52)$$

where m and e are the electronic mass and charge respectively, U_s is the orbital velocity of the s^{th} electron in the atom of the medium, $V_o = e^2/\hbar$, ze is the charge of the ion and n is the number of atoms of the medium per unit volume.

III.2.6 LSS Treatment

For the energy loss of charged particles in stopping media with atomic numbers less than 10, Lindhard et al.[11] worked out a theory based on the Thomas-Fermi statistical model of the atom. They expressed the energy E of the moving ion and the range R in terms of dimensionless parameters ϵ and ρ defined by

$$\epsilon = E\, a \frac{M_2}{Z_1 Z_2 e^2 (M_1 + M_2)} \qquad \ldots(3.53)$$

$$\text{and } \rho = R\, n\, M_2 \frac{4\pi a^2 M_1}{(M_1 + M_2)^2} \qquad \ldots(3.54)$$

where M_1, M_2 and Z_1, Z_2 are the mass numbers and the atomic

numbers of the moving ion and the stopping medium respectively, n is the number of atoms of the medium per unit volume, and a is the screening parameter and its value is given by

$$a = a_o \left[0.8853 \, (Z_1^{2/3} + Z_2^{2/3})^{-1/2} \right]$$

a_o being the Bohr radius of the hydrogen atom. The electronic stopping is assumed to be proportional to $\epsilon^{1/2}$:

$$(d\epsilon/d\rho)_e = K \epsilon^{1/2} \qquad \ldots(3.55)$$

$$\text{with } K = \xi_e \frac{0.0793 \, Z_1^{1/2} \, Z_2^{1/2} (M_1 + M_2)^{3/2}}{(Z_1^{2/3} + Z_2^{2/3})^{3/4} \, M_1^{3/2} \, M_2^{1/2}} \qquad \ldots(3.56)$$

and $\xi_e = Z_1^{1/6}$.

III.3 NUCLEAR COLLISIONS

The energy loss of heavy ions in elastic collisions with the atoms of the stopping medium takes place only near the end of the ion range, where the probability of electronic excitation becomes small. Bohr[3] showed that such two-body 'nuclear' collisions may be described classically if the collision diameter of the moving ion greatly exceeds the de Broglie wavelength. This condition is given by

$$\frac{Z_1 Z_2 e^2}{\hbar V} \gg 1 \qquad \ldots(3.57)$$

where Z_1 and Z_2 are the atomic numbers of the moving and struck particles respectively and V is the ion velocity.

To describe a 'nuclear' collision between two heavy ions, the Rutherford scattering law can be applied directly. In terms of energy transfer T, the differential cross section for collision between two particles is given by

$$d\sigma(E,T) = \frac{\pi Z_1^2 Z_2^2 e^4}{E} \left(\frac{M_1}{M_2}\right) \frac{dT}{T^2}, \qquad \ldots(3.58)$$

where M_1 and M_2 are the masses of the moving and struck particles. In the derivation of Eq. (3.58) it is assumed that the collision is occurring between two point charges with central fields of force. In the case of collisions between a heavy ion or a fission fragment and an atom of the stopping medium the presence of bound electrons on each nucleus shields the nucleus and in that case the simple Coulomb potential has to be modified to take care of the screening effect of the bound electrons. One such potential V(r) is given by

$$V(r) = (Z_1 Z_2 e^2/r) \exp(-r/a)$$

where a is a screening parameter and r is the distance. The applicability of the Rutherford law depends critically on the ratio between 'a' and the collision diameter 'b' for an

unscreened field $\xi = \frac{b}{a}$. If $\xi \ll 1$ or $b \ll a$ and also $\chi \gg 1$, then the enounter would be a very intimate one and the interaction would be confined mostly in the unscreened part of the field. For strong screening $\xi \gg 1$, a is small so that collisions may be considered as occurring between two rigid spheres. If R(E) is the hard sphere radii which depend on the energy, the differential scattering cross section is then given by

$$d\sigma(E,T) = 4\pi\left[R(E)\right]^2 (dT/T_{max}) \qquad \ldots(3.60)$$

where $T_{max} = \frac{4 M_1 M_2}{(M_1 + M_2)^2} E$

The differential cross section is related to the rate of energy loss by

$$-\frac{dE}{dx} = n \int_{T_{min}}^{T_{max}} T\, d\sigma(E, T) \qquad \ldots(3.61)$$

By a simple geometric treatment of the scattering process, Bohr[3] showed that

$$T_{min} = \frac{Z_1^2 Z_2^2 e^4}{a^2} \left(\frac{M_1}{M_2}\right) \frac{1}{E}, \qquad \ldots(3.62)$$

and the rate of energy loss due to nuclear collisions is given by

$$-\frac{dE}{dx} = \left(\frac{2\pi Z_1^2 Z_2^2 e^4}{M_2 V^2}\right) n \ln \frac{M_1 M_2}{M_1 + M_2} \frac{V^2 a_s}{Z_1 Z_2 e^2} \quad ..(3.63)$$

where a_s is the screening parameter and all the other symbols have been defined earlier.

CHAPTER IV

DEVELOPMENT OF RANGE-VELOCITY AND STOPPING-POWER EQUATIONS FOR FISSION-FRAGMENTS AND PARTIALLY STRIPPED HEAVY IONS

IV.1 DERIVATION OF THE RANGE-VELOCITY RELATION

The energy-loss per unit path length of a moving ion of charge ze and velocity V is given by Eq. (3.49).

$$-\frac{dE}{dx} = \frac{2\pi z^2 e^4 n}{mV^2}\left[\int_{U_s} \ln\left\{\eta_s^2 [\chi]^{-2}\right\} dn(U_s) + \int_{U_s} \ln\left\{\eta_s^2 \left[\frac{\chi}{\eta_s}\right]^{-1}\right\} dn(U_s)\right.$$

...(4.1)

The above equation differs from Eq.(3.49) only in the replacement of the summations by integrations. The integration should include the orbital velocities of only those electrons which interact with the moving ion at any given ion velocity V. Because of the continuous capture and loss of electrons by the decelerating ion its charge fluctuates and in that case ze in Eq.(4.1) has to be replaced by an 'effective charge' Z^{eff} of the ion. The evaluation of Z^{eff} and the integration of the

logarithmic terms were done by Bohr[3] in the case of a "heavy" ion passing through a "heavy" medium.

IV.1.1 Heavy solid media

Two assumptions were made by Bohr:[3]

(a) The number of orbital electrons $n(U_s)$, whose velocities are less than a given velocity U_s, is given by

$$n(U_s) = \nu_s \frac{U_s}{V_o} , \qquad ...(4.2)$$

where ν_s is the "effective" quantum number for the outer orbital electrons. In the case of heavy atoms ($Z \gtrsim 40$) the most firmly bound electrons, belonging to K, L, etc shells, move in a field which is approximately coulombic and have values of ν_s very nearly equal to 1, 2, ... etc, respectively. Furthermore the most loosely bound electrons for which $n(U_s) \sim 1$ again have values of ν_s of the order of unity. But over a large intermediate region, ν_s has a flat maximum corresponding to values close to $Z^{1/3}$, where Z is the atomic number of the atom. This result is in conformity with the analysis of electron binding by the Thomas-Fermi statistical model. $n(U_s)$ was expressed by Bohr[3] as

$$n(U_s) = Z^{1/3} \frac{U_s}{V_o} , \qquad ...(4.3)$$

holding for $V_o < U_s < Z^{2/3} V_o$. For values of U_s outside this region Eq.(4.3) is no longer valid and in that case $n(U_s)$ can be

represented by some function of Z and U_s, which for $U_s \sim V_o$ is of the order of unity, while it approaches Z for $U_s \sim ZV_o$.

(b) The second assumption, made by Bohr, is that the number of electrons lost by a heavy ion at a velocity V is equal to the number of orbital electrons with velocity less than V. Thus, Z^{eff} can be given directly from Eq.(4.3).

$$Z^{eff} = n(U_s = V) = Z^{1/3} \frac{V}{V_o} . \qquad ...(4.4)$$

The number $Z^{1/3}$ was estimated for the heavy fission fragments slowing down in gases.[30] The effective charge of the fragments in solids is difficult to evaluate, but it is expected to be higher because of much shorter time between collisions, compared to the time of de-excitation. Bohr and Lindhard[31] made a qualitative estimate of Z^{eff} which is given by

$$Z^{eff} = k\, Z^{1/3} \frac{V}{V_o} , \qquad ...(4.5)$$

where k = 1.5 in solid and k = 1 in gaseous media. The evaluation of two integrals of Eq. (4.1), was done with the help of Eq. (4.3). The lower limit was taken as $U_s = 0$ and the upper limit U_s' corresponds to those values of U_s which make the logarithmic terms zero. Considering these two integrals separately, one obtains

$$J_1 = \int_{U_s=0}^{U_s=U_s'} \ln\left\{\eta_s^2 [\chi]^{-2}\right\} dn(U_s) = \frac{Z_2^{1/3}}{V_o} \int_o^{U_s'} \ln\left\{\eta_s^2 [\chi]^{-2}\right\} dU_s$$

$$= 4\, Z_2^{1/3} \frac{V}{V_o}\, \chi^{-1}, \qquad \ldots(4.6)$$

and

$$J_2 = \int_{U_s=0}^{U_s=U_s'} \ln\left\{\eta_s^2 \left[\frac{\chi}{\eta_s}\right]^{-1}\right\} dn(U_s) \qquad \ldots(4.7)$$

To evaluate the above integral J_2 the integration has to be performed in two parts from 0 to $2V\chi^{-1}$ for $\frac{\chi}{\eta_s} < 1$ and $2V\chi^{-1}$ to U_s' for $\frac{\chi}{\eta_s} > 1$. Using these considerations in Eq.(4.7), J_2 becomes

$$J_2 = \frac{Z_2^{1/3}}{V_o} \left[\int_o^{2V\chi^{-1}} \ln\left\{\eta_s^2\right\} dU_s + \int_{2V\chi^{-1}}^{U_s'} \ln \eta_s^2 \left[\frac{\chi}{\eta_s}\right]^{-1} dU_s \right]$$

$$= \frac{Z_2^{1/3} V}{V_o} \left[\left\{4\chi^{-1} + 4\chi^{-1} \ln\chi\right\} + \left\{6\chi^{-1/3} - 6\chi^{-1} - 4\chi^{-1} \ln\chi\right\} \right]$$

$$= \frac{2Z_2^{1/3} V}{V_o} \left[3\chi^{-1/3} - \chi^{-1}\right] \qquad \ldots(4.8)$$

The sum of the integrals in Eq.(4.1) is given by

$$J = J_1 + J_2 = \frac{2Z_2^{1/3} V}{V_o} \left[3\chi^{-1/3} + \chi^{-1}\right] \qquad \ldots(4.9)$$

Use of Eq.(4.9) in Eq.(4.1) leads to the following energy loss equation

$$-\frac{dE}{dx} = \frac{4\pi e^4 N}{mV_o^3}\ \frac{\rho}{A_2}\ Z_2^{1/3}\left[4.7622(kZ_1^{1/3})^{5/3} + kZ_1^{1/3}\right]V \quad .. \quad (4.10)$$

Using Eq. (4.10) and $dE = A_1 m_o V dV$, where A_1 is the fragment mass number and m_o is the nucleonic mass and substituting the values of all the constants, one obtains the range-velocity equation

$$\rho\, dx = \frac{A_1\ A_2\ dV}{127.3 \times 10^{11}\ Z_2^{1/3}\left[4.7622(kZ_1^{1/3})^{5/3} + kZ^{1/3}\right]} \quad ...(4.11)$$

where A_1 and Z_1 are the mass number and atomic number of the moving ion while ρ, A_2 and Z_2 stand respectively, for the density, mass number and atomic number of the medium. The total range or "integrated range" might be obtained if one could integrate the right-hand side of Eq.(4.11) down to zero velocity. However, Eq.(4.11) is not strictly valid at $V \langle V_o$, since at these velocities the ion is practically neutral and stopping will be mainly due to screened field type of interactions. Niday[12] advanced the plausible argument that as far as the projected range in the direction of the moving ion is concerned, which corresponds to the experimentally measured range, it is sufficient to evaluate the integral between the initial ion velocity V_i and the velocity V_o at which the ion becomes almost neutral. At $V \langle V_o$ there would be insignificant addition to the projected range because of the very large angle scatterings

associated with nuclear collisions. Hence, the experimentally determined range may be related to the initial velocity V_i by the following expression:

$$R(mg/cm^2) = \int_0^R \rho dx = \frac{A_1 A_2 (V_i - V_o)}{127.3\, Z_2^{1/3}\left[4.7622(kZ_1^{1/3})^{5/3} + kZ_1^{1/3}\right]} \quad \ldots(4.12),$$

where both V_i and V_o are to be expressed in units of 10^8 cm/sec. and all other symbols have been defined earlier. Niday[12] showed that with $k \approx 1$, the ranges of various fission products in uranium can be calculated quite accurately. Since $k \approx 1$ is different from what Bohr and Lindhard[31] suggested, all available range data for fission products were analyzed in heavy solid media in order to investigate if there is any systematic, medium-dependent variation in the value of k. The procedure used for calculating the value of k is as follows. For a given fission product of mass number A_1 of known range R, the initial velocity V_i is first calculated. For this purpose, the precursor fragment mass number A_1' is first determined by means of the experimentally measured prompt-neutron number as a function of fragment mass number. From the experimentally measured fission fragment kinetic energies, existing in literature and A_1', the values of V_i have been calculated. Since the prompt neutrons are emitted almost isotropically from the moving fragments, the velocity of the fission product

of mass number A_1 is identical with the velocity of the precursor fragment of mass number A_1'. The atomic number Z_1 (also termed the nuclear charge) of the fission product of mass number A_1 is given by the "most-probable charge" $Z_p(A_1)$ which has been calculated from Mukherji's[32] prescription

$$\text{for light products} \qquad Z_p(A_1) = Z_c - \frac{A_c - A_1'}{2.587} \qquad \ldots(4.13)$$

$$\text{for heavy products} \qquad Z_p(A_1) = \frac{A_1'}{2.587} \qquad \ldots(4.14)$$

where A_c and Z_c are the mass and the atomic numbers of the parent compound fissile nucleus. Using appropriate values of V_i, Z_1 and A_1 in Eq.(4.12) the values of k for different fission products in a particular medium have been obtained. The range values have been taken from Niday[12] for uranium, Alexander and Gazdik[21] for Au, Almodovar et al.[33] and Hontzeas and Block[34] for tungsten and Smith and Frank[35] for zirconium. In all cases, the precursor fragment mass number A_1' has been obtained with the prompt neutron emission data of Apalin et al.[36] The fragment kinetic energies have been taken from Schmitt et al.[37] Table IV.1 shows the k values obtained with different fission products in uranium. For other heavy catchers like Zr, W and Au, k values are listed in Table IV.2. The arithmetic mean values of k obtained from Tables IV 1 and IV.2 in U, Zr, W and Au together with the

TABLE IV.1

Values of k calculated from Eq. (4.12) for different fission-products in uranium

Nuclides	Velocity (in units of 10^8 cm/sec)	Most probable charge(Z_p)	k
1	2	3	4
^{86}Rb	14.9407	37.00	1.0950
^{89}Sr	14.6398	35.46	1.0544
^{90}Sr	14.5750	35.95	1.0359
^{91}Sr-Y	14.4956	36.33	1.0551
^{93}Y	14.3405	37.11	1.0644
^{95}Zr	14.1478	37.92	1.0602
^{97}Zr	13.9922	38.75	1.0585
^{99}Mo	13.8829	39.54	1.0700
^{103}Ru	13.5270	41.20	1.0562
^{106}Ru	12.9561	42.42	1.0541
^{109}Pd	12.3880	43.68	1.0611
^{111}Ag	11.9988	44.45	1.0819
^{112}Pd	11.8298	44.98	1.0826
^{115}Cd	11.3200	46.09	1.0610
^{125}Sn	10.9786	48.49	1.0897
^{127}Sb-Te	10.9596	49.09	1.0607
$^{129}Te^{m}$	10.9296	49.98	1.0524
^{132}Te	10.6825	51.21	1.0492
^{137}Cs	9.9582	53.34	1.0347

contd.

Table IV.1(contd.)

1	2	3	4
^{140}Ba	9.6258	54.69	1.0457
^{141}Ce	9.4863	55.08	1.0498
^{143}Ce	9.1912	56.04	1.0383
^{144}Ce	9.0879	56.43	1.0346
^{147}Nd	8.9047	56.43	1.0307

TABLE IV.2

Values of k calculated from Eq.(4.12) for different fission products in Zr, W, and Au

Nuclides	$(k)_{Zr}$	$(k)_{W}$	$(k)_{Au}$
^{89}Sr	0.9739	-	0.9587
^{97}Zr	-	-	0.9637
^{99}Mo	1.0040	0.9962	-
^{111}Ag	-	-	0.9933
^{115}Cd	-	-	0.9858
^{131}I	-	-	0.9818
^{132}Te	-	1.0010	-
^{140}Ba	0.9876	1.0030	0.9637

TABLE IV.3

Mean values of k in different heavy media

Medium	Atomic number (Z_2)	k Mean
Zirconium	40	0.988 ± 0.05
Tungsten	74	1.000 ± 0.003
Gold	79	0.980 ± 0.03
Uranium	92	1.06 ± 0.03

maximum deviation of an individual value from the mean value are shown in Table IV.3. Since there are finite errors associated with the measured ranges, it is possibly justifiable to assume that in heavy solid media and possibly in all solid media k may be considered as unity. This analysis supports Bohr's original assumption that $Z^{eff} = Z^{1/3} \frac{V}{V_o}$, independent of the medium being penetrated. Hence the ranges in all heavy media can be obtained from the following expression:

$$R(mg/cm^2) = \frac{A_1 A_2 (V_i - V_o)}{127.3\ Z_2^{1/3}\left[4.7622\ Z_1^{5/9} + Z_1^{1/3}\right]} \quad \ldots(4.15)$$

IV.1.2 Light solid media

In the case of light solid media, the main problem lies in the integration of the logarithmic terms in Eq.(4.1).Since the minimum value of Z_1 in the case of fission products is close to 40, the use of the Thomas-Fermi approach is justifiable and Eq.(4.4) can be used to calculate the effective charges. However, for media like Be, C, Al and Si ($Z<40$), Eq.(4.3) is certainly not applicable, since no statistical approach can be valid for systems containing only a few electrons. Bohr[3] used Eq.(4.3) to evaluate the summation terms of Eq.(3.49) for heavy media. In our analysis, we have proceeded with the assumption that in the case of light atoms ($Z<40$) one may take $n(U_s)$, the number of electrons having velocity

less than a given velocity U_s to be

$$n(U_s) = f(Z) \frac{U_s}{V_o} , \qquad \ldots(4.16)$$

where f(Z) is a function of Z whose form has to be determined. With this assumption and following Bohr's[3] procedure of replacing the summation terms in Eq.(3.49) with integrals, and making appropriate provision for the non-participation of many of the orbital electrons of the light media, one gets an expression analogous to Eq.(4.9) for the integral J.

$$J = 2f(Z_2) \frac{V}{V_o} \left[3\bar{\chi}^{1/3} + \bar{\chi}^{1} \right] \qquad \ldots(4.17)$$

After substitution of the numerical values of the physical constants and the replacement of the integrals in Eq.(4.1) by Eq.(4.17), the energy loss equation becomes

$$\frac{dE}{dx} \left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = 1.327 \frac{f(Z_2)}{A_2} \left[4.7622\ Z_1^{5/9} + Z_1^{1/3} \right] V \qquad \ldots(4.18).$$

Further using $dE = A\,V dV$ and Niday's[12] assumption for the lower cut-off velocity V_o, the range-velocity equation becomes

$$R(\text{mg/cm}^2) = \frac{A_1 A_2 \ (V_i - V_o)}{127.3\ f(Z_2) \left[4.7622\ Z_1^{5/9} + Z_1^{1/3} \right]} \qquad \ldots(4.19)$$

In order to evaluate $f(Z_2)$, the same procedure has been followed as in the case of heavy media for evaluating k. Range data for various fission products in a particular medium along with the corresponding values of V_i and $Z_1\left[Z_p(A_1)\right]$ were put in Eq.(4.19) to obtain individual values of $f(Z_2)$ in one medium. Ranges in Al were taken from Alexander and Gazdik,[21] Aras et al.,[38] Brown and Oliver,[16] and Nakahara et al.,[39] in carbon from Chinaglia et al.,[19] and in Be and Si from Demichelis et al.[40] Only one datum on range in Cu is available from Segre' and Wiegand[17] for the average fission product from the thermal neutron fission of ^{235}U. Since the fission products in equal and maximum yields have mass and atomic numbers 140, 99 and 54.70, 39.44 respectively, the values of A_1 and Z_1 for the average fission product may be taken as 119.5 and 47.07 respectively. Using similarly an average velocity V_i, $f(Z_2)$ for Cu has been calculated from Eq. (4.19). Table IV.4 shows the values of $f(Z_2)$ obtained for different fission products in Be, C, Al and Si. The arithmetic mean value of $f(Z_2)$ together with its maximum deviation from an individual value for any particular medium is shown in Table IV.5. Zirconium[35] has also been included among the light media to locate the dividing line between the "light" and the "heavy" media. The plot of $f(Z_2)$ vs $Z_2^{2/3}$, shown in Fig. 1, shows a slope of 0.28 and thus $f(Z_2)$ may be very

TABLE IV.4

$f(Z_2)$ Values for Be, C, Al, and Si obtained from Eq.^{2}(4.19) using different fission products

Nuclides	Be	C	Al	Si
^{87}Rb	-	-	1.6019	-
^{89}Sr	-	-	1.5390	-
^{91}Sr	-	-	1.5190	-
^{92}Y	-	-	1.5775	-
^{95}Zr	0.7412	0.9110	-	1.6153
^{97}Zr	-	-	1.5728	-
^{99}Mo	0.7149	0.9007	1.5815	1.6035
^{103}Ru	0.7185	0.9546	-	1.5794
^{111}Ag	-	-	1.5937	-
^{115}Cd	-	-	1.5850	-
^{131}I	0.7016	0.9103	1.5709	1.6008
$^{132}Te-I$	0.6823	0.8660	-	1.5768
^{137}Cs	-	-	1.6190	-
^{140}Ba	0.7221	0.8502	1.6088	1.6148
^{141}Ce	0.6953	-	-	-
^{143}Ce	-	0.8253	-	1.5911
^{147}Nd	0.7043	0.8128	-	-

TABLE IV.5

Mean $f(Z_2)$ values for different light media

Medium	Atomic number (Z_2)	$f(Z_2)$
Hydrogen	1	0.275 ± 0.012
Helium	2	0.347 ± 0.009
Beryllium	4	0.710 ± 0.03
Carbon	6	0.879 ± 0.07
Nitrogen	7	1.080 ± 0.02
Neon	10	1.166 ± 0.02
Aluminium	13	1.582 ± 0.02
Silicon	14	1.597 ± 0.02
Argon	18	2.182 ± 0.04
Copper	29	2.64
Zirconium	40	3.23 ± 0.33

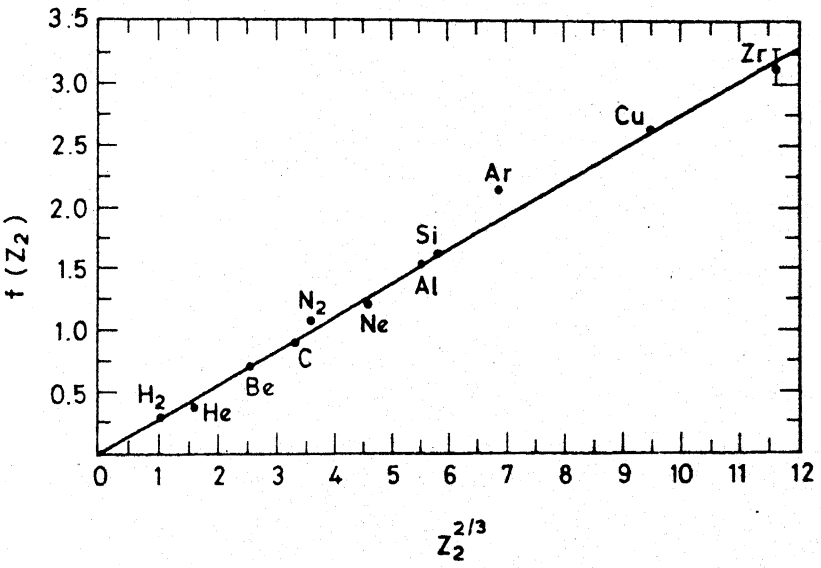

FIG. 1. Plot of $f(Z_2)$ vs $Z_2^{2/3}$ for various media.

accurately given by

$$f(Z_2) = 0.28\ Z_2^{2/3} \qquad \ldots(4.20)$$

Further, from $n(U_s) = Z_2^{1/3} \frac{U_s}{V_o} = 0.28\, Z_2^{2/3} \frac{U_s}{V_o}$, it is found that $Z_2 = 45.5$ constitutes the dividing line between the light and the heavy media. Thus, the number of electrons $n(U_s)$ having velocity less than a given velocity U_s in the case of media with $Z < 45.5$ is given by

$$n(U_s) = 0.28\ Z_2^{2/3} \frac{U_s}{V_o} \qquad \ldots(4.21)$$

Similarly, the effective charge of an ion of velocity $V (Z < 45.5)$ is given by

$$Z^{eff} = 0.28\ Z^{2/3} \frac{V}{V_o} \qquad \ldots(4.22)$$

Eqs. 4.15, 4.19 and 4.21 lead to the following comprehensive range-velocity equation in solid media:

$$R(mg/cm^2) = \frac{A_1 A_2 (V_i - V_o)}{127.3\ f(Z_2) \left[4.7622 \left\{f(Z_1)\right\}^{5/3} + f(Z_1)\right]} \qquad \ldots(4.23)$$

where A_1, A_2 and Z_1, Z_2 are the mass numbers and the atomic numbers of the fission product (or heavy ion) and the medium respectively, V_i is the initial velocity of the moving ion and $V_o = 2.18 \times 10^8$ cm/sec. Both V_i and V_o are to be expressed in units of 10^8 cm/sec.; $f(Z_1)$ or $f(Z_2)$ or in general $f(Z)$ is

given by

$$f(Z) = 0.28\ Z^{2/3} \quad \text{for} \quad Z \lesssim 45.5 \qquad \ldots(4.24)$$

$$f(Z) = Z^{1/3} \quad \text{for} \quad Z \gtrsim 45.5 \qquad \ldots(4.25)$$

IV.1.3 Light gaseous media

Range data for fission products from the fission of ^{233}U in the light gases H_2, He, N_2, Ne and Ar have been taken from Pertzhak et al.[41] and the $f(Z_2)$ values were calculated using Eq.(4.19). The initial velocity V_i and the precursor fragment mass number A_1' have been calculated with the data of Pleasonton[42] and Apalin et al.[36]. Table IV.6 shows the results of the calculation for $f(Z_2)$ for different gases.

TABLE IV.6

Values of $f(Z_2)$ for gases obtained from Eq.(4.19)

Nuclides	Hydrogen	Helium	Nitrogen	Neon	Argon
^{91}Sr	0.2788	0.3545	1.0791	1.1635	2.1478
^{92}Y	0.2784	0.3558	1.1075	1.1556	2.1619
^{97}Zr	0.2864	0.3512	1.0986	1.1790	2.2060
^{140}Ba	0.2740	0.3485	-	-	2.2220
^{143}Ce	0.2555	0.3261	1.0552	-	2.1764

The arithmetic mean values of $f(Z_2)$ thus obtained are shown in Table IV.5. From Fig. 1, it is evident that except for Ar, $f(Z_2)$ for other gases (H_2, He, N_2, Ne) are accurately given by Eq. (4.20). In the case of hydrogen and helium this agreement may be fortuitous, but this would be useful in calculating the ranges in compound media like mylar and collodion, which contain a number of hydrogen atoms, as shown in Sec. IV.1.5.

IV.1.4 Ranges of accelerated heavy ions

Although there should not be any difference between a fission product and an accelerated heavy ion as far as the applicability of Eq.(4.23) is concerned, it would be interesting to verify this. Bridwell and Moak[22] have measured the ranges of ^{127}I and ^{79}Br ions accelerated to different energies in the form of R(E)-R(10), where R(E) and R(10) are the ranges corresponding to energies E(MeV) and 10 MeV respectively. Figure 2 shows these values along with our calculated values for the four elements beryllium, aluminium, nickel and gold. Hower and Fairhall[43] have measured the energy degradation of ^{9}Be ion during its passage through different thickness of aluminium and gold. In the case of gold the calculated value of the range R(6.8 MeV)-R(2.2 MeV) is 4.504 mg/cm^2, whereas the experimental value is found to be 4.50 mg/cm^2, R(6.8 MeV) and R(2.2 MeV) being the ranges corresponding to initial

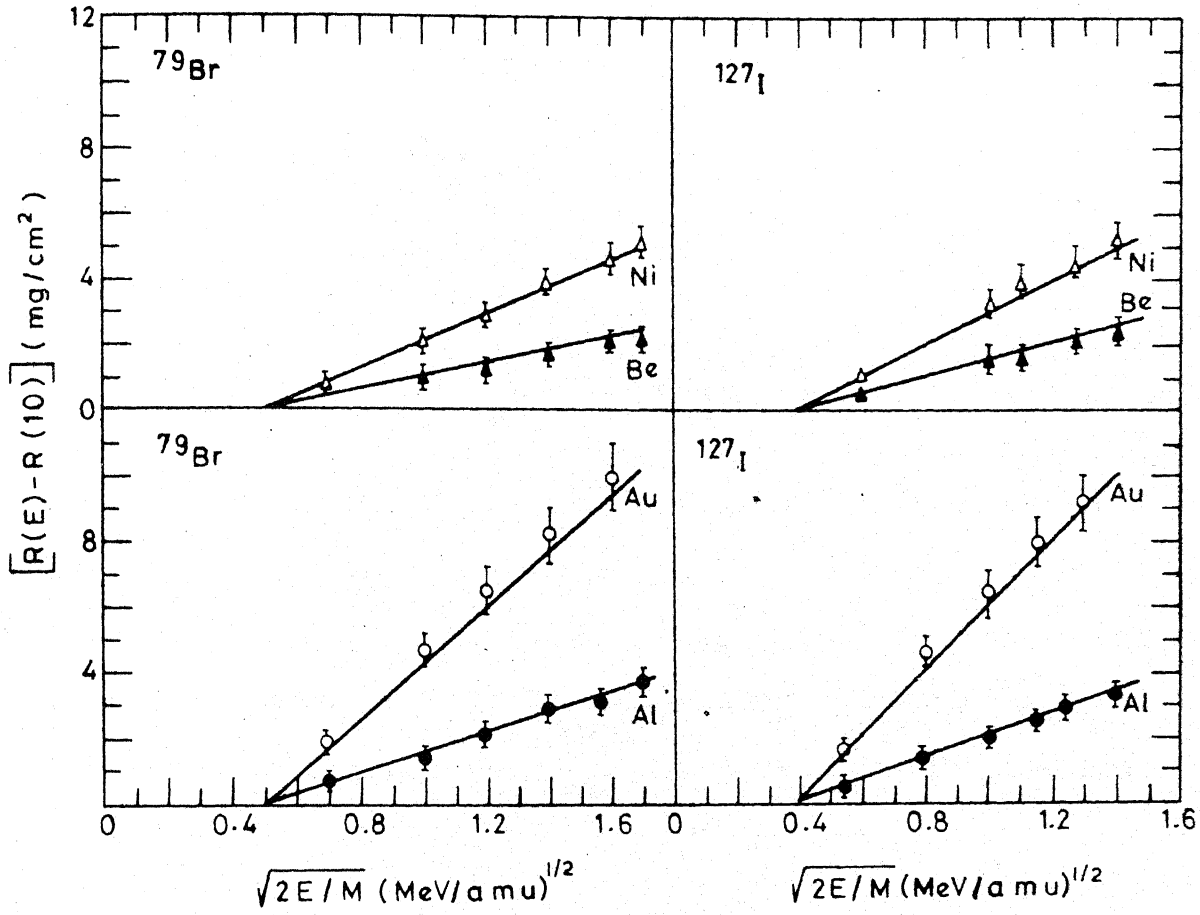

FIG. 2. Plot of $R(E)-R(10)$ against $(2E/M)^{1/2}$ for ^{79}Br and ^{127}I ions in Be, Al, Ni, and Au. $R(E)$ and $R(10)$ are the ranges corresponding to initial energies E MeV and 10 MeV, respectively. $\bar{\Delta}$ and $\bar{\bigcirc}$, experimental values from Bridwell and Moak, Ref. 22; —, calculated values.

energies of 6.8 MeV and 2.2 MeV. In the case of aluminium no such reliable informations could be extracted from the experiment.

IV.1.5 Compound media

The stopping power equation given by Eq.(4.18) may be written as

$$-\frac{dE}{dx} = \alpha\, n\, f(Z_2)\left[4.762\left\{f(Z_1)\right\}^{5/3} + f(Z_1)\right]V, \quad \ldots(4.26)$$

where α is a constant and n is the number of atoms of the medium per unit volume and $f(Z_1)$ and $f(Z_2)$ or $f(Z)$ is given by Eqs. (4.24) and (4.25). If a compound is made up of X_i number of atoms of the i^{th} atomic species per molecule, then the actual stopping power would be the sum of the stopping powers due to all the individual atoms in each molecule.Hence,

$$-\frac{dE}{dx} = \alpha\,\frac{\rho N}{A_c}\left\{\sum_i X_i f(Z_i)\right\}\left[4.7622\left\{f(Z_1)\right\}^{5/3} + f(Z_1)\right]V, \quad \ldots(4.27)$$

where ρ is the density of the medium, N is the Avogadro number, A_c is the mass number of the molecule $A_c = \sum_i X_i A_i$; Z_i and A_i being the atomic number and the mass number of the i^{th} atomic species. Hence, for a compound medium, the stopping-power and range-velocity equations are

$$-\frac{dE}{dx} = 1.327\,\frac{f(Z_c)}{A_c}\left[4.7622\left\{f(Z_1)\right\}^{5/3} + f(Z_1)\right]\,V, \quad \ldots(4.28)$$

$$R = \frac{A_1 A_c \, (V_i - V_o)}{127.3 \, f(Z_c) \left[4.7622 \left\{ f(Z_1) \right\}^{5/3} + f(Z_1) \right]} \quad , \ldots (4.29)$$

where $f(Z_c) = \sum_i X_i \, f(Z_i)$ and $A_c = \sum_i X_i A_i$ and $f(Z)$ is defined by Eqs. (4.24) and (4.25). The calculation of ranges in mylar ($C_{10}H_8O_4$), collodion ($C_{12}H_{17}O_{16}N_3$) and UF_4 would involve some assumptions regarding $f(Z_i)$. For carbon and uranium as also for hydrogen, $f(Z_i)$ is known. Since nitrogen also conforms to $f(Z) = 0.28 \, Z^{2/3}$, it is assumed that this would also be true in the cases of oxygen and fluorine. Using the appropriate mass and atomic numbers and the initial velocity for the average fission product from the thermal neutron fission of ^{235}U as indicated in Sec. IV.1.1, range in collodion has been calculated by means of Eq. (4.28) and is shown in Table IV.7, along with the corresponding experimental value of Segre' and Wiegand.[17] The calculated and the experimental ranges for the "average" fission product in aluminium are also shown in the same table as an additional verification of the correctness of our procedure for calculating A_1, Z_1, and V_i of the "average" fission product. Table IV.7 also lists the calculated values of the ranges of fission products of mass number 99 and 140 in CsI and mylar along with the corresponding experimental values of Suzor[44] and Cumming and Crespo.[8] Figure 3 shows the plot of the calculated values of

TABLE IV.7

Calculated and experimental ranges in mg/cm^2 in compound media

Fission product	Collodion		Mylar		Al		CsI	
	Cal.	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.	Exp.
Average fission product A_1 = 119.5 Z_1 = 47.07 V_i = 11.75	2.609	2.6[a]	-	-	3.65	3.70[a]	-	-
Median light A_1 = 99 Z_1 = 39.44	-	-	2.756	2.57±0.05[b]	-	-	7.84	7.6[c]
Median heavy A_1 = 140 Z_1 = 54.70	-	-	2.20	2.11±0.04[b]	-	-	6.08	6.6[c]

a, Segre' and Wiegand[17]

b, Cumming and Crespo[8]

c, Suzor[44]

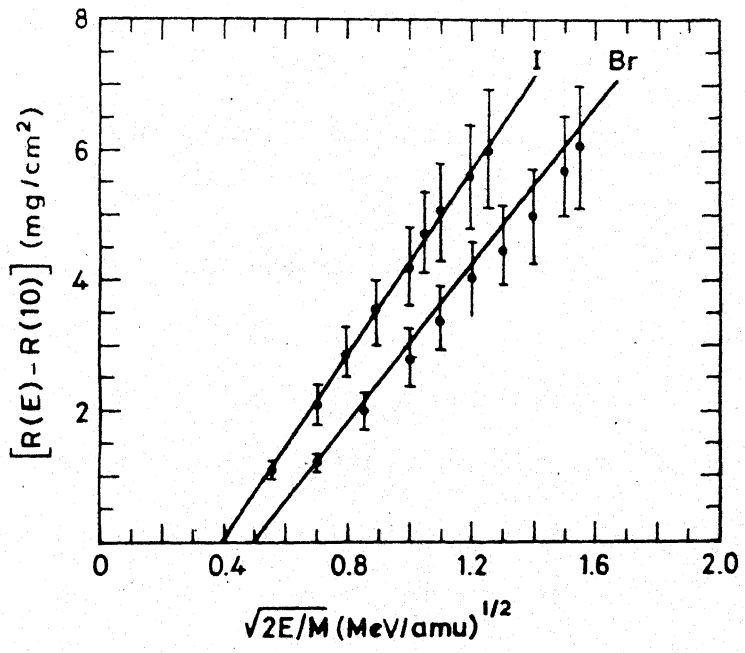

FIG. 3. Plot of $R(E)$-$R(10)$ vs $(2E/M)^{1/2}$ for ^{79}Br and ^{127}I ions in UF_4. $R(E)$ and $R(10)$ are the ranges corresponding to initial energies E MeV and 10 MeV, respectively. The symbols with the error bars show the experimental values, while the straight lines represent the calculated values (——).

$R(E)-R(10)$ in UF_4 vs $(2E/M)^{1/2}$ along with the corresponding experimental values[22] for ^{79}Br and ^{127}I ions, where $R(E)$ and $R(10)$ correspond to ranges at initial ion energy of E and 10 MeV respectively, and M is the mass number of the heavy ion.

IV.2 STOPPING-POWERS

Niday's[12] assumption, which has been used in the derivation of the range-velocity relation that at $V<V_o$ the penetrating ion does not travel much in its original direction, needs an independent verification. The best procedure would be to determine whether the stopping powers given by Eq.(4.1), after the incorporation of the phenomenological deductions represented by Eqs. (4.16), (4.24) and (4.25), does predict the stopping powers in different media at different energies which are in agreement with the available experimental data. The comprehensive stopping-power equation is

$$-\frac{dE}{dx}\left(\frac{\text{MeV-cm}^2}{\text{mg}}\right) = 1.327\,\frac{f(Z_2)}{A_2}\left[4.762\left\{f(Z_1)\right\}^{5/3} + f(Z_1)\right]V \quad ..(4.30)$$

where V is the velocity of the ion in units of 10^8 cm/sec at which the stopping power is being calculated and all other symbols have been defined earlier. In the case of a compound of known molecular formula, the observed stopping power would

be the sum of the stopping powers of the individual atoms in the molecule and is given by Eq.(4.28). Comparison of the experimental and calculated values of the stopping powers are shown in the following sections.

IV.2.1 Fission fragments

Figure 4 shows the stopping powers of different fission products from the spontaneous fission of ^{252}Cf calculated with Eq. (4.30) along with the corresponding experimental values of Bridwell and Walters[10] (using thick foils of silver and gold). All calculated values are at $V = 1.38 \times 10^9$ cm/sec in the case of carbon, $V = 1.35 \times 10^9$ cm/sec in the case of silver and $V = 1.32 \times 10^9$ cm/sec in the case of gold. The experimental errors, which are of the order of 25-30% in the cases of silver and gold and is somewhat higher in the case of carbon, are not shown in the figure. Kahn and Forgue[9] have measured the energy degradation of the median light and the median heavy fission products from the spontaneous fission of ^{252}Cf and the thermal-neutron fission of ^{235}U, using foils of different metals of varying thicknesses. Using these authors' values for the mass and atomic numbers of the fission products, the experimental stopping powers have been obtained in the following manner. From the incident and emergent energies E_i and E_e of the fission products before and after

passing through a foil of thickness 't', the experimental stopping power is obtained as $(E_i - E_e)/t$. The corresponding $(\frac{dE}{dx})$ is calculated from Eq. (4.30) using a value of V corresponding to the mean energy $\frac{1}{2}(E_i + E_e)$. Figures 5 and 6 show these values in aluminium, nickel, silver and gold. In a similar manner the energy-loss data of Müller and Gönnenwein[24] have been treated to obtain the experimental and calculated stopping powers for the median light and median heavy fission products from the spontaneous fission of ^{252}Cf. Figure 5 includes these results only in the case of copper; for the other metallic media the agreement between the experimental and calculated values are excellent but are not shown in the figure. Only one experimental datum on $(\frac{dE}{dx})$ for the median light fission product from ^{252}Cf could be obtained from the data of Kahn and Forgue[9] in the case of U-Pd alloy, which is shown in Table IV.8. The alloy containing 20-wt.% U and 80-wt.% Pd has the molecular formula $U\text{-}Pd_{8.88}$ and using Eq.(4.28), the stopping power has been calculated and is shown in Table IV.8.

IV.2.2 Accelerated heavy ions

Table IV.9 shows the experimental values of the stopping-powers of accelerated ^{14}N ion as measured by Nakata[45] along with the corresponding calculated values with Eq.(4.30) in aluminium, nickel, silver, and gold.

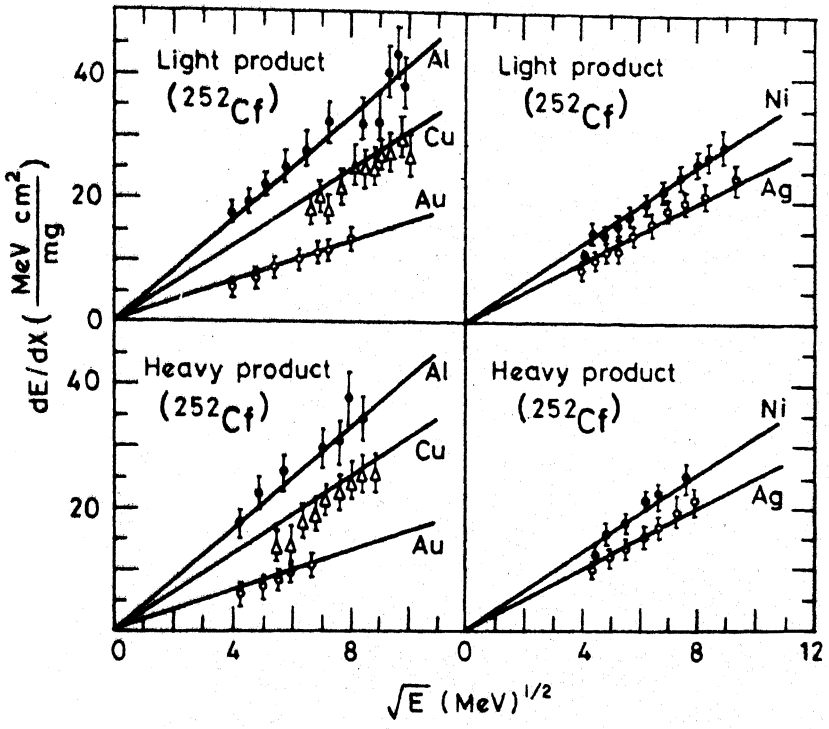

FIG. 5. Plot of the stopping powers (dE/dX) vs $\sqrt{E}$ for the median heavy and median light fission products from the spontaneous fission of ^{252}Cf in Al, Ni, Cu, Ag, and Au. The symbols with the error bars show the experimental values, while the straight lines represent the calculated values. The experimental values in Al, Ni, Ag, and Au are from Kahn and Forgue (Ref. 9) and in Cu from Müller and Gönnenwein (Ref. 24).

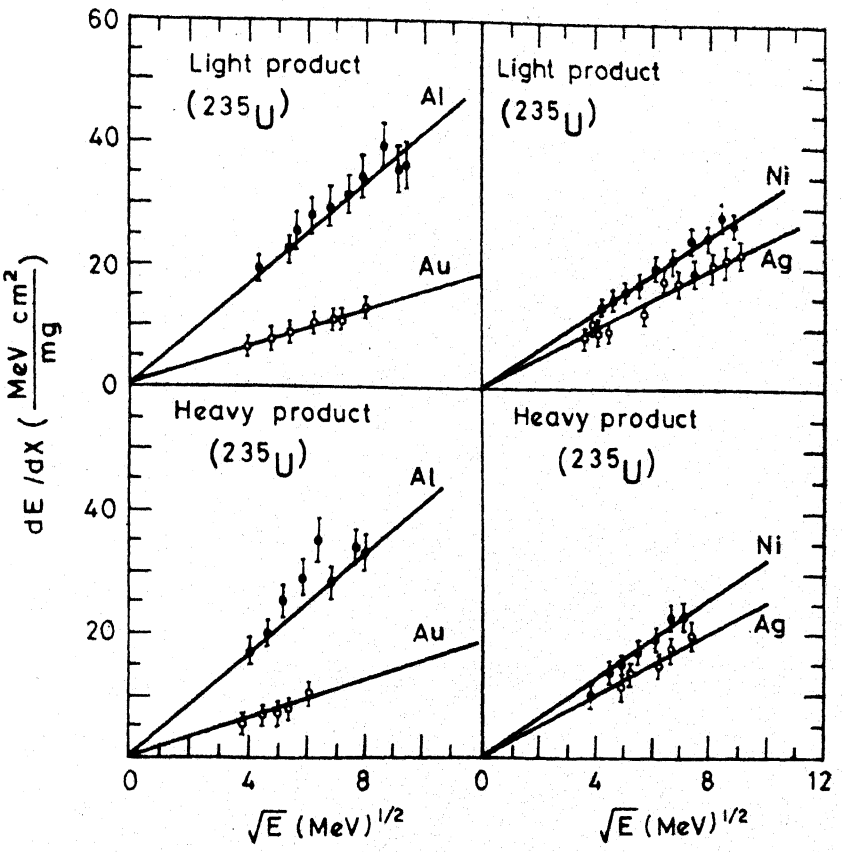

FIG. 6. Plot of the stopping powers (dE/dX) vs $\sqrt{E}$ for the median heavy and median light fission products from the thermal neutron fission of ^{235}U in Al, Ni, Ag, and Au. The symbols with the error bars show the experimental values taken from Kahn and Forgue (Ref. 9) and the straight lines represent the calculated values.

TABLE IV.8

Stopping-power of the median light fission product from the spontaneous fission of ^{252}Cf in U-Pd alloy. The experimental value is from Kahn and Forgue[9]

A_1	Z_1	t(mg/cm^2) (foil thickness)	E_i (MeV)	E_e (MeV)	ΔE (MeV)	$\frac{(E_i + E_e)}{2}$ (MeV)	$\frac{\Delta E}{\Delta x}$ ($\frac{\text{MeV cm}^2}{\text{mg}}$) (exp.)	$\frac{\Delta E}{\Delta x}$ ($\frac{\text{MeV cm}^2}{\text{mg}}$) (calc.)
106.44	42.55	0.73	34.2	25.7	8.5	29.9	11.64	12.27

TABLE IV.9

Calculated and experimental values[45] of the stopping powers of ^{14}N ion in Al, Ni, Ag and Au

Stopping medium	Mean energy (MeV)	$(dE/d)_{Cal.}$ (MeV cm^2/mg)	$(dE/dx)_{Exp.}$ (MeV cm^2/mg)	% Uncertainty in exp. (dE/dx)
Al	8.00	5.23	5.00	5.59
	6.90	4.85	4.40	6.04
Ni	6.65	3.59	3.60	6.28
	4.06	2.80	2.75	8.11
Ag	5.50	2.53	2.65	6.85
Au	5.85	1.69	1.85	6.53
	6.75	1.82	2.00	6.16

Only two values of the stopping-powers in the cases of aluminium, nickel and gold are shown in Table IV.9, while in the case of silver only one datum could be obtained from the experiment. The overall uncertainty in $(\frac{dE}{dx})$ has been obtained from the errors involved in the measurement of the foil thickness and the energies of the incident and emergent ions. Shane and Seaman[46] measured the stopping powers of 18.4 MeV and 16.9 MeV ^{20}Ne ion in aluminium using a new technique in which the Doppler shift of γ-rays from a moving ion is used to

measure the velocity of the ion. Table IV.10 shows the calculated values of the stopping powers with the corresponding experimental values.

TABLE IV.10

Calculated and experimental values[46] of the stopping powers of ^{20}Ne ion in aluminium

Energy (MeV)	(dE/dx) Cal. (MeV cm^2/mg)	(dE/dx) Exp. (MeV cm^2/mg)
18.4	9.46	9.4±0.6
16.9	9.05	10.6±1.8

Figures 7 and 8 show the experimental values of the stopping powers of accelerated ^{127}I and ^{79}Br ions as measured by Moak and Brown[47] at various energies along with the corresponding calculated values with Eq.(4.30) in beryllium, carbon, aluminium, nickel, silver and gold. In Fig. 9 the values of the stopping powers of ^{127}I and ^{79}Br ions in UF_4 are shown along with the corresponding measured values of Bridwell and Moak.[22] Figure 10 represents the experimental values of the stopping powers of accelerated ^{238}U ion in carbon, aluminium, nickel, silver and gold from Brown and Moak[48] and the corresponding values calculated with Eq. (4.30). The experimental values have ±10% error which are not shown in the figure.

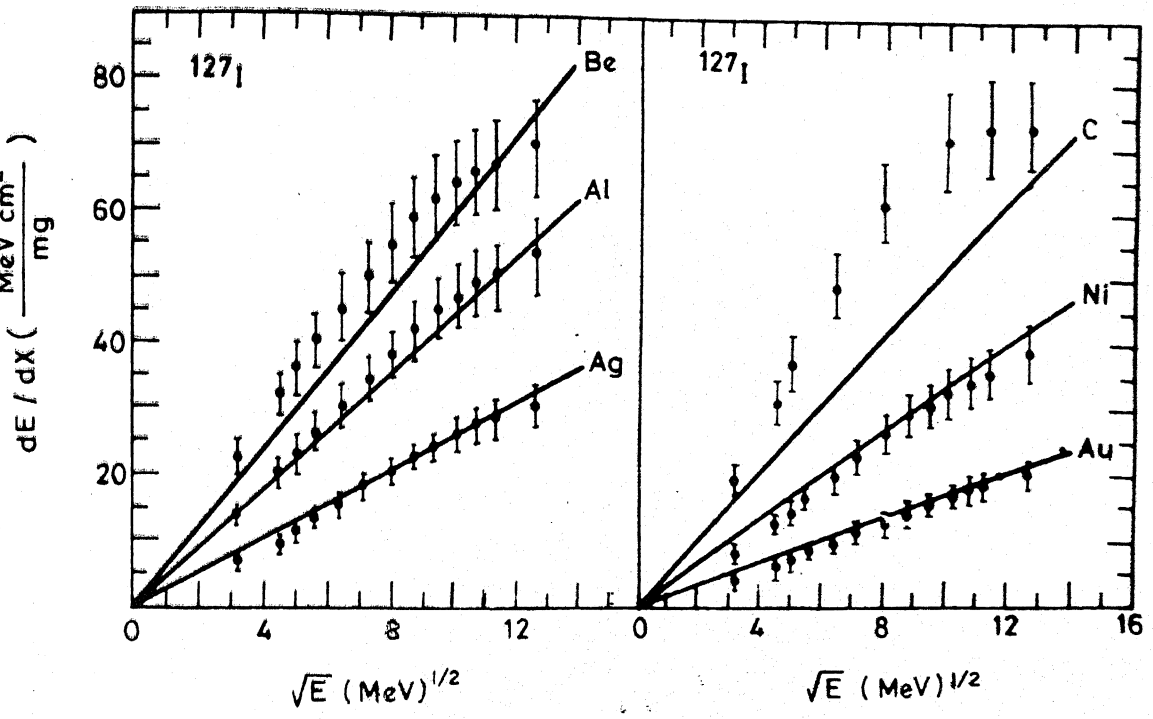

FIG. 7. Plot of the stopping powers (dE/dX) of ^{127}I ions vs $\sqrt{E}$ in Be, C, Al, Ni, Ag, and Au. The experimental values from Moak and Brown (Ref. 47) are shown by the symbols with the error bars, while the straight lines represent the calculated values.

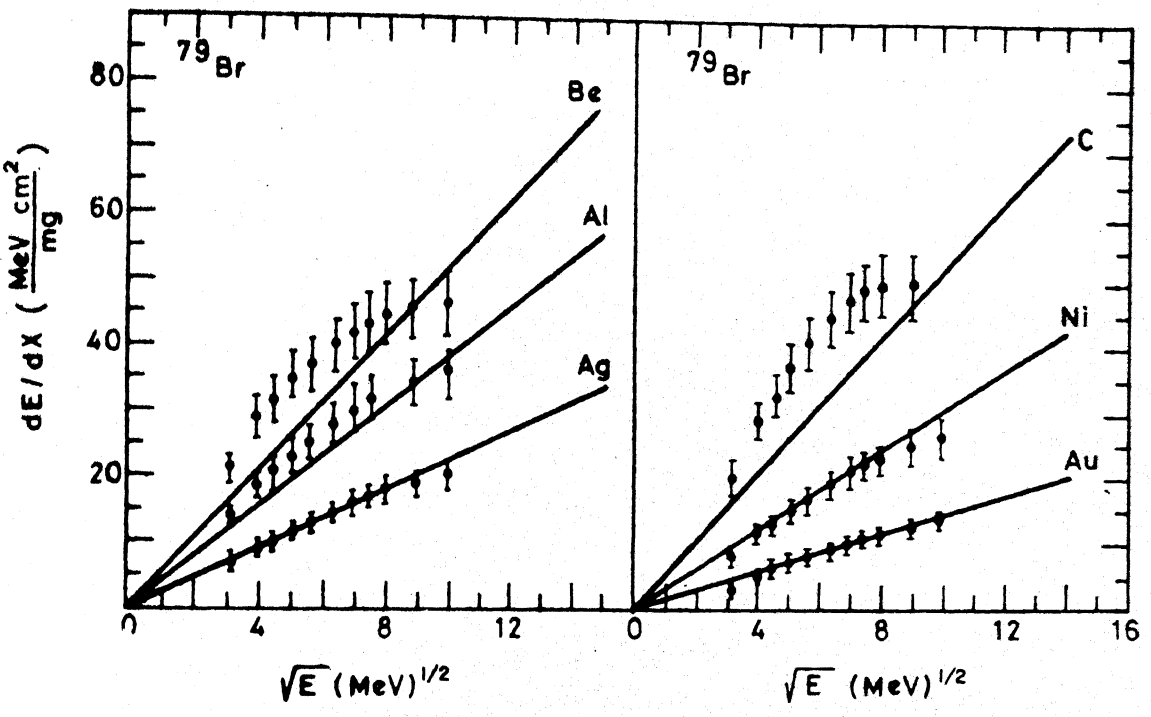

FIG. 8. Plot of the stopping powers (dE/dX) of ^{79}Br ions vs $\sqrt{E}$ in Be, C, Al, Ni, Ag, and Au. The experimental values from Moak and Brown (Ref.47) are shown by the symbols with the error bars, while the straight lines represent the calculated values.

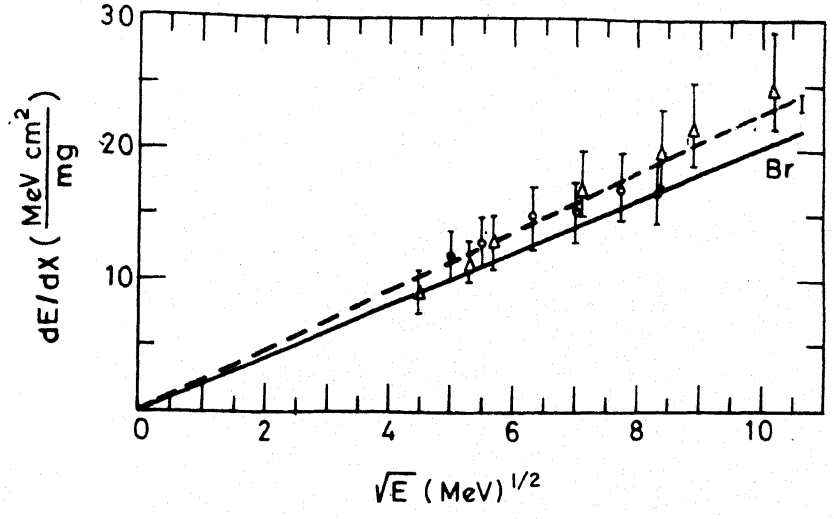

FIG. 9. Plot of the stopping powers (dE/dX) of ^{79}Br and ^{127}I ions vs $\sqrt{E}$ in UF_4. The experimental points from Bridwell and Moak (Ref. 22) are shown by the symbols $\overline{\Delta}$ for ^{127}I and $\overline{\bigcirc}$ for ^{79}Br. The calculated values are shown by the straight lines (—— for ^{79}Br and --- for ^{127}I).

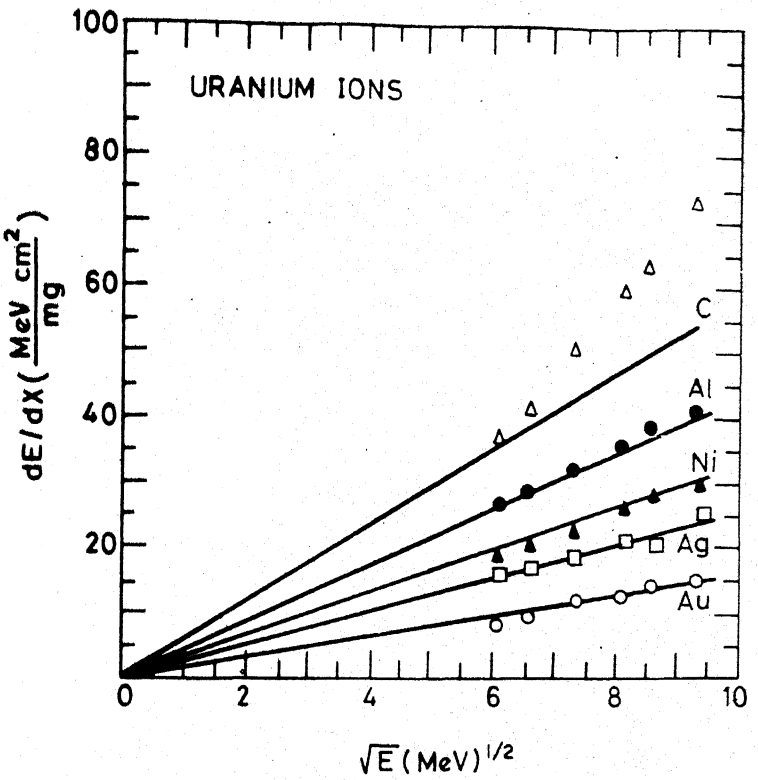

FIG. 10. Plot of the stopping powers (dE/dX) of ^{238}U ions vs $\sqrt{E}$ in C, Al, Ni, Ag, and Au. The symbols show the experimental values from Brown and Moak (Ref.48) and the straight lines show the corresponding calculated values. The experimental errors (±10%) are not shown.

IV.3 SIMPLER FORM OF STOPPING-POWER EQUATION

A simpler, but somewhat less precise, stopping power equation can be obtained from Bohr's[2] classical equation for the energy-loss of a swiftly moving charged particle i.e., Eq.(3.6):

$$-\frac{dE}{dx} = \frac{4\pi e^4 (Z^{eff})^2}{mV^2} n Z_2 \ln \frac{1.123\, mV^2}{Z^{eff} e^2 \bar{\omega}} , \qquad ...(4.31)$$

where $\bar{\omega}$ represents the geometric mean value of the cyclic frequency of the electron in an atom of atomic number Z_2 averaged over all the orbital electrons. Because of the non-participation of many of the orbital electrons of a heavy medium in interactions with partially stripped heavy ions and because of a continuous increase in the number of such non-participating electrons with decreasing ion velocity, it is not meaningful to use Eq.(4.31) with $\bar{\omega}$ in the logarithmic term. Instead, one may retain the summation sign:

$$-\frac{dE}{dx} = \frac{4\pi e^4 (Z^{eff})^2 n}{mV^2} \sum_s \ln \frac{1.123\, mV^3}{Z^{eff} e^2 \omega_s} , \qquad ...(4.32)$$

where the summation has to be carried out over all the s orbital electrons of the medium and ω_s is the cyclic frequency of the s^{th} orbital electron. The logarithmic term may be simplified by putting $Z^{eff} = f(Z_1) \frac{V}{V_o}$, $I_s = \hbar \omega_s = \frac{1}{2} m U_s^2$,

and $V_o = e^2/\hbar$, where e and m are the electronic charge and mass, I_s is the ionization potential of the s^{th} electron with orbital velocity U_s and n is the number of atoms of the medium per unit volume. Following Bohr's[3] procedure, the summation may be replaced by integration, with the upper limit $U_s = U_s'$, where U_s' makes the logarithmic term zero. The logarithmic term in Eq. (4.32) is

$$\sum_s \ln\left\{\frac{1.123\, mV^3}{Z^{eff} e^2 \omega_s}\right\} = \sum_s \ln\left\{\frac{2(1.123)V^2}{f(Z_1)\, U_s^{\,2}}\right\} = \int_{U_s=0}^{U_s=U_s'} \left\{\ln \frac{2.246V^2}{f(Z_1)\, U_s^{\,2}}\right\} dn(U_s)$$

$$= \frac{2f(Z_2)}{V_o}\left\{\left(\frac{2.246}{f(Z_1)}\right)^{1/2} V\right\} \qquad \ldots(4.33)$$

Eq. (4.33) when used in Eq. (4.32) leads to the following energy loss equation:

$$-\frac{dE}{dx} = \frac{8(2.246)^{1/2}\pi e^4 n}{m V_o^{\,3}} f(Z_2)\left\{f(Z_1)\right\}^{3/2} V , \qquad \ldots(4.34)$$

which on integration gives the following range-velocity relation

$$R = \frac{A_1 A_2 (V_i - V_o)}{127.3\, f(Z_2)\left\{6\left[f(Z_1)\right]^{3/2}\right\}} , \qquad \ldots(4.35)$$

in which all the symbols are identical with those in Eq.(4.23) Since Eq.(4.23) predicts heavy ion ranges with accuracy, a comparison between Eqs. (4.23) and (4.35) shows that the latter would be useful if

$$\frac{4.7622\left[f(Z_1)\right]^{5/3}+f(Z_1)}{6\left[f(Z_1)\right]^{3/2}}\sim 1 \qquad \ldots(4.36)$$

Actual computation of the above ratio shows that for all ions with $30\langle Z_1\langle 45.5$ it remains close to 1.05 while for $Z_1\rangle 45.5$ it varies from 1.055 to 1.07. Thus, adjustments of Eqs. (4.34) and (4.35) by a factor of 1.06 leads to the following simple equations, which are about $\pm 1\%$ less precise than Eqs. (4.23) and (4.30):

$$-\frac{dE}{dx}=8.4\,\frac{f(Z_2)}{A_2}\left[f(Z_1)\right]^{3/2}\,V \qquad \ldots(4.37)$$

$$R=\frac{A_1A_2(V_i-V_o)}{809f(Z_2)\left[f(Z_1)\right]^{3/2}} \qquad \ldots(4.38)$$

Further Eqs. (4.23) and (4.30) may be somewhat simplified by noting the following:

$$\left\{4.7622\left[f(Z_1)\right]^{5/3}+f(Z_1)\right\}\approx 20+0.5\,Z_1 \quad \text{for } Z_1\rangle 45.5 \qquad \ldots(4.39)$$

$$\left\{4.7622\left[f(Z_1)\right]^{5/3}+f(Z_1)\right\}\approx 0.938\cdot Z_1 \quad \text{for } Z_1\langle 45.5 \qquad \ldots(4.40)$$

with the introduction of ~1% error.

IV.4 DISCUSSION

IV.4.1 Limits of Applicability

Equations (4.23) and (4.30) are based on the integrations, represented by Eq. (4.17), performed with $\chi > 1$. For partially stripped ions this imposes the condition;

$$2 Z^{eff} \frac{V_o}{V} > 1 \qquad \ldots(4.41)$$

or

$$2 f(Z_1) > 1$$

for the applicability of Eqs. (4.23) and (4.30). On the other-hand, Eqs. (4.37) and (4.38), based on purely classical treatment,[2] require $\chi \gg 1$ or $2 f(Z_1) \gg 1$ for their validity. Only in the case of ^{238}U ion for which $2 f(Z_1) \approx 9$, this condition may be considered as fulfilled. However, the calculated ranged of light fission products as also the stopping powers of ^{79}Br ion agree with the corresponding experimental data and $Z_1 = 30$ has been taken as the lower limit for the use of classical Eqs. (4.37) and (4.38). For ions like ^{9}Be and ^{14}N, classical equations do not hold since for these ions the condition $\chi \gg 1$ is not fulfilled. For light ions (^{9}Be , ^{14}N) the condition represented by Eq. (4.41) is satisfied and Eqs. (4.23) and (4.30) are valid. Thus, $Z_1 = 3$, where χ is just greater than unity, has been taken for the validity of Eqs. (4.23) and (4.30).

In the case of the media, the lower limit of Z_2 are obtainable from Eqs.(4.7) and (4.33). If U_s' is the value of U_s which makes the logarithmic term zero and if the orbital velocity of the K-shell electrons in the atom of the medium is U_K, then for $U_s' \gg U_K$ Eq.(4.7) loses its significance. Use of this criterion requires that

$$U_K \gg 2\, V\, \chi^{-1/3} \qquad \ldots(4.42)$$

This condition is obtained from the second integral of Eq. (4.7), i.e.,

$$\int_{U_s=2V\bar{\chi}^{-1}}^{U_s'} \ln\left\{ \eta_s^{\,3}\, \chi^{-1} \right\} dn(U_s) = 3 \int_{U_s=2V\bar{\chi}^{-1}}^{U_s'} \ln\left\{ \frac{2V\, \bar{\chi}^{1/3}}{U_s} \right\} d(U_s)$$

From the above equation the logarithmic term vanishes if $U_s = 2V\bar{\chi}^{1/3}$ and this leads to the condition given by Eq. (4.42). Using $\chi = 2f(Z_1)$ in Eq.(4.42) one obtains

$$V \ll U_K \left[\frac{1}{4}\, f(Z_1) \right]^{1/3}, \qquad \ldots(4.43)$$

as essential condition for the validity of Eqs. (4.23) and (4.30). Similarly the classical treatment would be valid if

$$U_K \gg \left\{ \frac{2.246}{f(Z_1)} \right\}^{1/2} \cdot V \qquad \ldots(4.44)$$

This leads to the following condition regarding the ion velocity V :

$$V \ll U_K \left\{ \frac{f(Z_1)}{2.246} \right\}^{1/2} \qquad \ldots(4.45)$$

Eq.(4.44) is barely satisfied in the case of fission products in carbon as the medium. The fit of our range-velocity equation in the case of Be may be either fortuitous or due to the relatively small contribution of the higher values of U_s to the total integral. In the case of hydrogen and helium the agreement appears to be purely fortuitous. In the case of heavy media, U_K may greatly exceed the ion velocity at which the ion becomes completely stripped of its orbital electrons. Bohr's limitation[31] that Z^{eff} should always be less than $\frac{Z_1}{2}$ for the applicability of Eqs. (4.23) and (4.30) does not seem to apply here. The assumption $Z^{eff} < \frac{Z_1}{2}$ requires that

$$V \ll \frac{Z_1 V_o}{2} \left\{ f(Z_1) \right\}^{-1}, \qquad \ldots(4.46)$$

where V is the velocity of the ion, Z_1 its nuclear charge and $f(Z_1)$ has been defined earlier. The inequality given by Eq. (4.46) is not always fulfilled. There are velocity regions in which the ion is partially stripped with $\chi > 1$ but its velocity may exceed $\frac{Z_1 V_o}{2f(Z_1)}$.

IV.4.2 Deviation from Niday's approach

Niday,[12] as a first approximation, integrated Eq. (4.11) between the limits V_i and V_o on the assumption that at $V \leq V_o$ the moving ion undergoes large-angle scatterings and the resultant penetration in the beam direction is negligible. Niday observed slightly lower ranges in uranium by using lead rather than aluminium to catch the fragments. To explain this discrepancy Niday assumed that significant scattering occurs along the path of the fragments in heavy media at V_c, the ion velocity at which the collision diameter becomes equal to the screening radius and it was taken as the lower velocity at which the nuclear collision becomes predominant. V_c is obtained from the relation

$$\frac{2Z_1Z_2e^2}{\mu V_c^2} = \frac{a_o}{(Z_1^{2/3} + Z_2^{2/3})^{1/2}}, \qquad \ldots(4.47)$$

where Z_1 and Z_2 are the nuclear charges of the ion and the stopping atoms, μ is the reduced mass of the system and a_o is the 'first Bohr radius' of the hydrogen atom. Using V_c as the lower limit for the velocity, one gets from Eq. (4.11).

$$R(mg/cm^2) = \frac{A_1A_2(V_i - V_c)}{127.3\, f(Z_2)\left[4.7622\left\{kZ_1^{1/3}\right\}^{5/3} + kZ_1^{1/3}\right]} \qquad \ldots(4.48)$$

Calculations with V_c as the lower limit, using the above equation, show that the values of k vary from 1.07 to 1.12 as one goes from the heavy to the light media (i.e., from uranium to carbon and beryllium). Using the new value of k instead of k=1 in Eq. (4.5), it is found that in most of the cases (Al, Cu, Ag, Ni and Au) the agreement between the calculated and the experimental stopping powers tends to become worse but gives a better agreement between the calculated and the experimental values of the stopping powers in carbon and beryllium. To optimize the agreement between the calculated and the experimental values of both ranges and stopping powers, V_o has been taken as the lower velocity limit for the range calculation.

IV.4.3 Stopping powers in carbon and beryllium

The calculated stopping powers in carbon and beryllium have been consistently lower than the corresponding experimental values. On the other hand, the ranges of fission products in both carbon and beryllium, calculated with Eq. (4.23), are in good agreement with the corresponding experimental values. No attempt has been made to explain this inconsistency but both Bridwell and Moak[22] and Booth and Grant[49] have mentioned that oxidation during thin film preparation by vacuum evaporation in the cases of both beryllium

and carbon and the tenacious retention of moisture in the case of carbon may lead to unassignable errors in the measured stopping powers.

IV.4.4 Range in the most favoured direction of a single crystal

If one assumes that electronic collision is the only form of interaction by which the ion loses its energy till it stops, which may be approximately true in the "most favoured" direction in a single crystal, then the range-velocity equation, Eq. (4.23), should be modified to the following form in the case of a single crystal

$$R(mg/cm^2) = \frac{A_1 A_2 V_i}{127.3\, f(Z_2)\left[4.7622\left\{f(Z_1)\right\}^{5/3} + f(Z_1)\right]} \qquad \ldots(4.49)$$

Hence, for the same ion or fission product, the ratio of the ranges in polycrystal R_{poly} and a mono-crystal R_{mono} (in the most favoured direction) would be given by the ratio of Eqs. (4.23) and (4.49).

$$\left[\frac{R_{mono}}{R_{poly}}\right]_{cal} = \left[\frac{V_i}{V_i - V_o}\right] \qquad \ldots(4.50)$$

The validity of Eq. (4.49) can be checked by comparing the ratio $\frac{R_{mono}}{R_{poly}}$ obtained from Eq. (4.50) and from the experimentally measured ranges in the most favoured direction of a single crystal R_{mono} and in the poly crystal R_{poly}.

CHAPTER V

STOPPING-POWER EQUATIONS FOR COMPLETELY STRIPPED HEAVY IONS

Bethe's energy loss equation is based on the assumption that $\chi \ll 1$ i.e.,

$$2z\frac{V_o}{V} \ll 1, \qquad \ldots(5.1)$$

a condition which is always fulfilled in the case of low mass ions i.e., protons and alpha particles, above 0.4 MeV/amu and Bethe's equation [Eq.(3.12)] can be used for stopping power calculations. In the case of heavy ions Eq.(3.12) can be used if the velocity of the ion is high enough to satisfy Eq.(5.1), but in practice this condition is not always fulfilled. There are certain velocity regions in which $\chi > 1$ and in this particular velocity domain neither Bethe's equation[4] nor Bohr's classical equation[2] is applicable. Eq. (4.30) which was derived for the partially stripped ions with $\chi > 1$ does not hold for completely stripped ions even though χ may be greater than unity.

It would be shown in this chpater that by a careful choice of the limits of integrations appearing in Eq. (4.1), it is possible to calculate the stopping powers of heavy ions, including proton, at an energy which is high enough for electronic collisions to be the predominant mode of energy loss. The derivation, calculation, and discussion of these are presented in the following sections.

V.1 DERIVATION

V.1.1 Energy-loss equations

The logarithmic terms in Bohr's energy-loss equation, Eq. (4.1), may be written in the form of integrals as discussed in Chapter IV.

$$J = \int_{U_s=0}^{U_s=U_s'} \ln\left\{\eta_s^2\left[\chi\right]^{-2}\right\} dn(U_s) + \int_{U_s=0}^{U_s=U_s'} \ln\left\{\eta_s^2\left[\frac{\chi}{\eta_s}\right]^{-1}\right\} dn(U_s), \qquad \ldots(5.2)$$

where $\eta_s = \frac{2V}{U_s}$ and $\chi = 2z\frac{V_o}{V}$. The second integral in the above equation may be evaluated in two parts, as shown in Sec.IV.1.1 of Chapter IV, corresponding to the limits within which $(\frac{\chi}{\eta_s})$ is less than and greater than unity respectively. Indicating the three integrals by J_1, J_2, and J_3, one gets

$$J = J_1 + J_2 + J_3, \qquad \ldots(5.3)$$

where $$J_1 = \int_{U_s=0}^{U_s=U_s'} \ln\left\{\eta_s^2 [\chi]^{-2}\right\} dn(U_s) \qquad \ldots(5.4)$$

$$J_2 = \int_{U_s=0}^{U_s=2V\chi^{-1}} \ln\left\{\eta_s^2\right\} dn(U_s) \quad \text{for } \frac{\chi}{\eta_s} < 1 \qquad \ldots(5.5)$$

and $$J_3 = \int_{U_s=2V\chi^{-1}}^{U_s=U_s''} \ln\left\{\eta_s^2 \left(\frac{\chi}{\eta_s}\right)^{-1}\right\} dn(U_s) \qquad \ldots(5.6)$$

Each integral requires careful examination for fixing the upper limit of integration corresponding to different actual physical situations. Two cases may be considered depending on the values of χ.

(A) $\chi > 1$

(a) $V > \frac{Z_2 V_o}{2}\chi$

Since $\eta_s = \frac{2V}{U_s}$, the first integral J_1, may be written as

$$J_1 = \int_{\text{all } U_s} \ln\left\{\left(\frac{2V}{U_s}\right)^2 \chi^{-2}\right\} dn(U_s), \qquad \ldots(5.7)$$

where the upper limit of the integration is given by that value of U_s for which the logarithmic term vanishes. This puts the condition $2V > U_s\chi$ for the logarithmic term to be positive. Further, since the maximum value which U_s can assume corresponds to the orbital velocity of the K-shell electrons of the medium, the logarithmic term would be always positive if

$$V \rangle \frac{Z_2 V_o}{2} \chi \quad , \qquad \ldots(5.8)$$

where the velocity of the K-shell electrons is assumed to be $Z_2 V_o$. Putting $I_s = \frac{1}{2} m U_s^2$, in Eq.(5.7), J_1 thus may be written as

$$J_1 = \int_{\text{all } U_s} \ln \left(\frac{2mV^2}{I_s \chi^2}\right) dn(U_s)$$

$$= Z_2 \ln \frac{2mV^2}{\bar{I} \chi^2} \quad , \qquad \ldots(5.9)$$

where I_s is the ionization potential of the s^{th} orbital electron of the medium of atomic number Z_2 and $\bar{I}$ stands for the geometric mean ionization potential of the medium, obtained by averaging over <u>all</u> the orbital electrons.

$$Z_2 \ln \bar{I} = \int_{\text{all } U_s} \ln I_s \, dn(U_s) \qquad \ldots(5.10)$$

As far as the integral J_2 is concerned, the logarithmic term $\left\{\ln\left(\frac{2V}{U_s}\right)^2\right\}$ would be positive for all values of U_s upto $Z_2 V_o$, if $V \rangle \frac{Z_2 V_o}{2}$. Since, the condition $V \rangle \frac{Z_2 V_o}{2} \chi$ has already been assumed, $V \rangle \frac{Z_2 V_o}{2}$ is automatically satisfied for $\chi \rangle 1$. Using $I_s = \frac{1}{2} m U_s^2$ and Eq.(5.10), in Eq. (5.5), J_2 may be given by

$$J_2 = \int_{\text{all } U_s} \ln \left\{\left(\frac{2V}{U_s}\right)^2\right\} dn(U_s)$$

$$= Z_2 \ln \frac{2mV^2}{\bar{I}} \qquad \ldots(5.11)$$

Further, since the upper limit for U_s in J_2 is $2V\chi^{-1}$, the third integral J_3 becomes superfluous for $2V\chi^{-1} \gg Z_2V_o$, since the lower limit for the integral J_3 already corresponds to the maximum possible value for U_s. Thus, the combination of J_1 and J_2, given by Eqs.(5.9) and (5.11), and substitution in Eq. (4.1) lead to the following expression for the energy loss:

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mV^2} n Z_2 \ln \frac{2mV^2}{\bar{I}\chi}, \qquad \ldots(5.12)$$

where all the symbols have been defined earlier.

(b) $\frac{Z_2V_o}{2}\chi \gg V \gg \frac{Z_2V_o}{2}\chi^{1/3}$

Since $\chi \gg 1$, $\frac{Z_2V_o}{2}\chi \gg \frac{Z_2V_o}{2}\chi^{1/3} \gg \frac{Z_2V_o}{2}$, and J_1, J_2 and J_3 are evaluated as follows:

J_1 requires an upper cut-off value $U_s = U_s'$, since the logarithmic term $\left\{\ln \left(\frac{2V}{U_s}\right)^2\right\}$ would otherwise assume negative values for some values of $U_s \ll Z_2V_o$. Hence from Eqs.(5.4) and $dn(U_s) = f(Z_2)\frac{dU_s}{V_o}$,

$$J_1 = \int_{U_s=0}^{U_s'} \left\{\ln\left(\frac{2V}{U_s\chi}\right)^2\right\} dn(U_s) = -\frac{2f(Z_2)}{V_o}\int_0^{U_s'} \ln\left\{\left(\frac{U_s}{2V\chi}\right)\right\}^{-1} dU_s$$

$$= \frac{4f(Z_2)V}{V_o\chi} \qquad \ldots(5.13)$$

Because of the condition that $2V\chi^{-1} < Z_2V_o$, the second integral J_2, given by Eq. (5.5) does not include all possible values of U_s upto Z_2V_o:

$$J_2 = \int_o^{2V\chi^{-1}} \ln\left\{\left(\frac{2V}{U_s}\right)^2\right\} dn(U_s) = -\frac{2f(Z_2)}{V_o} \int_o^{2V\chi^{-1}} \ln\left(\frac{U_s}{2V}\right) dU_s$$

$$= \frac{4f(Z_2)V}{V_o\chi}(1+\ln\chi), \quad ..(5.14)$$

The third integral J_3 is given by

$$J_3 = \int_{U_s=2V\chi^{-1}}^{U_s=U_s'} \ln\left\{\left(\frac{2V}{U_s}\right)^3 \chi^{-1}\right\} dn(U_s)$$

$$= \frac{3f(Z_2)}{V_o} \int_{2V\chi^{-1}}^{U_s'} \ln\left\{\left(\frac{2V}{U_s}\right)\chi^{-1/3}\right\} dU_s \quad ..(5.15)$$

The above integral vanishes for $U_s = 2V\chi^{-1/3}$. Since the maximum value of U_s is assumed to be Z_2V_o, if Z_2V_o exceeds $2V\chi^{-1/3}$ then an upper cut-off value has to be used for evaluating the integral J_3, because for $2V\chi^{-1/3} \gg Z_2V_o$ it is physically meaningless to evaluate the integral in the domain $2V\chi^{-1/3} \gg U_s \gg Z_2V_o$. The upper cut-off value for the third integral may be obtained from a consideration of the number $n(U_s)$ of orbital electrons with $U_s > 2V\chi^{-1}$ which interact with

the ion at a given velocity V. In the actual evaluation of J_3 it is convenient to consider the contribution of the two K-shell electrons separately from that of the other electrons with $U_s \rangle 2V\chi^{-1}$. This procedure avoids the complications arising out of the relativistic velocities of the K-shell electrons in heavy media as well as the inaccuracy involved in using the expression for $n(U_s)$, given by Eq. (4.16), in the case of the inner-most electrons. Eq.(4.16) for $n(U_s)$ suffers from the drawback that it does not lead to the correct velocities of either the innermost or the outermost electron. Further, the range and the stopping power data, on which the validity of Eq. (4.16) is based involve only a few outer electrons in the case of heavy media and may not hold for the deeper-seated electrons. Taking these limitations into consideration, J_3 may be written as

$$J_3 = \int_{U_s=2V\chi^{-1}}^{U_s=\frac{(Z_2-2)V_o}{f(Z_2)}} \left\{\ln\left(\frac{2V}{U_s}\right)^3 \chi^{-1}\right\} dn(U_s) + 2\ln\left\{\left(\frac{2V}{Z_2V_o}\right)^3 \chi^{-1}\right\}$$

$$= -\frac{3f(Z_2)V}{V_o} \int_{2V\chi^{-1}}^{\frac{(Z_2-2)V_o}{f(Z_2)}} \left\{\ln\left(\frac{U_s}{2V}\right) \chi^{1/3}\right\} dn(U_s) + 2\ln\left\{\left(\frac{2V}{Z_2V_o}\right)^3 \chi^{-1}\right\}$$

$$= \left[3(Z_2-2)\ln\frac{2Vf(Z_2)}{V_o(Z_2-2)}\chi^{-1/3} + 3(Z_2-2) + \frac{3f(Z_2)}{V_o}\right.$$

$$\left.\left\{2V\chi^{-1}(\ln\chi^{-2/3}-1)\right\} + 6\ln\frac{2V}{Z_2V_o} - 2\ln\chi\right] \quad \ldots(5.16)$$

J is given by the sum of Eqs. (5.13), (5.14) and (5.16) which, when incorporated into Eq. (4.1), gives

$$-\frac{dE}{dx} = \frac{2\pi z^2 e^4 n}{mV^2}\left[3(Z_2-2)+3(Z_2-2)\ln\frac{2Vf(Z_2)}{(Z_2-2)V_o}+6\ln\frac{2V}{Z_2V_o}+\frac{2f(Z_2)V}{V_o\chi}-Z_2\ln\chi\right], \quad \ldots(5.17)$$

where $\chi = 2z\frac{V_o}{V}$ and all the other symbols have been defined earlier.

(c) $V < \frac{Z_2V_o}{2}\chi^{1/3}$

If $V < \frac{Z_2V_o}{2}\chi^{1/3}$ then both J_1 and J_3 would require appropriate cut-off velocities U_s', since all the orbital electrons can not have interactions with the moving ion. This case has already been considered in Sec. IV.1.1 of Chapter IV. The energy loss is given by

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mV^2} n \frac{f(Z_2)V}{V_o}\left[3\chi^{-1/3}+\chi^{-1}\right] \quad \ldots(5.18)$$

(B) For $\chi < 1$

For $\chi < 1$, since $[\chi]^{-2}$ is unity in the integral J_1, both the integrals J_1 and J_2 become identical. Thus, J is given by

$$J = J_1 + J_2 = 2 \int_{\text{all } U_s} \ln \eta_s^2 \, dn(U_s) \qquad \ldots(5.19)$$

The third integral J_3 does not come into the picture, because for $\chi < 1$, $(\frac{\chi}{\eta_s})$ is always less than unity, as shown in Sec.III.2.5 of Chapter III. From Eq. (5.19)

$$J = 2 \int \ln \left(\frac{2V}{U_s}\right)^2 dn(U_s)$$

$$= 2Z_2 \ln \frac{2mV^2}{\bar{I}} \qquad \ldots(5.20)$$

where $\bar{I}$ is given by Eq. (5.10). Thus, the total energy loss becomes,

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4 n}{mV^2} Z_2 \ln \frac{2mV^2}{\bar{I}} \qquad \ldots(5.21)$$

This equation is identical with Bethe's equation except that $\bar{I}$ stands for the mean ionization potential as given in Eq. (5.21), while that in Bethe's equation $\bar{I}$ stands for the mean excitation potential. Bohr[3] showed that the mean excitation potential can be put equal to the mean ionization potential of the electrons with sufficient accuracy.

V.1.2 Energy-loss equations for completely stripped heavy ions

Except the proton and the alpha particle which have $\chi < 1$ under all conditions as mentioned earlier, all other

heavy ions would have $\chi > 1$ in the incompletely stripped state, may have $\chi > 1$ at certain velocities in the completely stripped state and at appropriate high velocities would have $\chi < 1$. Since completely stripped ions are considered, the condition which should be fulfilled is that at the minimum velocity, the ion should have its charge number $z(Z^{eff})$ equal to the nuclear charge Z_1. Hence,

$$Z^{eff} = f(Z_1)\ \frac{V}{V_o} = Z_1\ , \qquad \ldots(5.22)$$

where $f(Z_1)$ is given by Eqs.(4.24) and(4.25) and $V_o = e^2/\hbar$. This gives

$$V > \frac{Z_1 V_o}{f(Z_1)}\ , \qquad \ldots(5.23)$$

as the essential condition for the ion being completely stripped. For such ions the energy loss equations are given by Eqs.(5.12), (5.17), (5.18) and (5.21) with $z = Z_1$. After substitution of the values of the physical constants in the energy loss equations, one obtains the following simplified relations:

(A) $\chi > 1$ or $V < 2Z_1 V_o$

(a) $V > \frac{Z_2 V_o}{2}\chi$ or $V > \sqrt{Z_1 Z_2}\, V_o$

$$-\frac{dE}{dx}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = 63.65\ \frac{Z_1^2 Z_2}{A_2 V^2}\ \log\left(\frac{11.39V^2}{\bar{I}\chi}\right), \qquad \ldots(5.24)$$

(b) $\frac{Z_2 V_o}{2}\chi \gg V \gg \frac{Z_2 V_o}{2}\chi^{1/3}$ or $\sqrt{Z_1 Z_2}V_o \gg V \gg \frac{Z_1^{1/4} Z_2^{3/4} V_o}{\sqrt{2}}$

$$-\frac{dE}{dx}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = 13.79 \frac{Z_1^2}{A_2 V^2}\left[3(Z_2-2) + 3(Z_2-2)\ln\frac{2Vf(Z_2)}{(Z_2-2)V_o} + 6\ln\frac{2V}{Z_2 V_o} + \frac{2f(Z_2)V}{V_o \chi} - Z_2 \ln\chi\right] \quad \ldots(5.25)$$

(c) $V \ll \frac{Z_2 V_o}{2}\chi^{1/3}$ or $V \ll \frac{Z_1^{1/4} Z_2^{3/4} V_o}{\sqrt{2}}$

$$-\frac{dE}{dx}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = 12.68 \frac{f(Z_2)}{A_2}\frac{Z_1^2}{V}\left[3\chi^{-1/3} + \chi^{-1}\right] \quad \ldots(5.26)$$

(B) For $\chi \ll 1$ or $V \gg 2Z_1 V_o$

$$-\frac{dE}{dx}\left(\frac{\text{MeV cm}^2}{\text{mg}}\right) = 63.65 \frac{Z_1^2 Z_2}{A_2 V^2}\log\left(\frac{11.39V^2}{\bar{I}}\right), \quad \ldots(5.27)$$

where $\bar{I}$ is the mean ionization potential in units of eV, V is the velocity of the ion and V_o is the electron velocity in the first Bohr orbit, both in units of 10^8 cm/sec; $f(Z_2)$ is given by Eqs.(4.24) and (4.25) and $\chi = 2Z_1 \frac{V_o}{V}$.

V.1.3 Range-energy relationship

The total range may be obtained from the energy loss equation by simple integration. Two cases arise depending on the values of χ.

(A) $\chi > 1$

For $\chi > 1$ there are three energy-loss equations, which are valid only in certain energy domains. Using $E = \frac{1}{2} M_1 V^2$, $dE = M_1 V dV$ and $\chi = 2z \frac{V_o}{V}$, in Eq. (5.12), one gets

$$dR = dx = \left(\frac{m}{4\pi e^4}\right) \frac{M_1 V^3 dV}{nz^2 Z_2 \ln\left(\frac{mV^3}{z V_o \bar{I}}\right)} \quad , \qquad \ldots(5.28)$$

where $M_1 = A_1 m_o$, m_o being the nucleonic mass. This equation is not valid at velocities given by $mV^3 \langle zV_o\bar{I}$, since the logarithmic term in Eq. (5.12) becomes negative and loses its significance. Therefore Eq. (5.28) can not be integrated over entire velocity range. But it can be used to describe the difference in range $(R_2 - R_1)$ between any two high velocities V_2 and V_1. Eq.(5.28) may be written as

$$dR = \left(\frac{m}{4\pi e^4}\right) \frac{A_1 m_o V^3 \, dV}{nz^2 Z_2 \frac{3}{2} \ln\left[\left(\frac{m}{z V_o \bar{I}}\right)^{2/3} V^2\right]} \quad , \qquad \ldots(5.29)$$

Substituting

$$C = \left[\left(\frac{m}{z V_o \bar{I}} \right)^{2/3} V^2 \right]^2, \qquad \ldots(5.30)$$

in Eq.(5.29) and integrating between the velocities V_1 and V_2, one obtains

$$\int_{R_1}^{R_2} dR = \left(\frac{m}{12 \pi e^4} \right) \frac{A_1 m_o}{n z^2 Z_2} \left(\frac{z V_o \bar{I}}{m} \right)^{4/3} \int_{C_1}^{C_2} \frac{dC}{\ln C} \qquad \ldots(5.31)$$

$$\text{or } (R_2 - R_1) = \left(\frac{m}{12\pi e^4} \right) \left(\frac{V_o}{m} \right)^{4/3} \frac{A_1 m_o}{n\, z^{2/3} Z_2} \bar{I}^{4/3} \left[Ei(\ln C_2) - Ei(\ln C_1) \right], \qquad \ldots(5.32)$$

where Ei stands for the exponential integral and C_1, C_2 or C is given by Eq. (5.30). Eq. (5.32) holds for all velocities $V \gg \left(\frac{z V_o \bar{I}}{m} \right)^{1/3}$.

(B) $\chi < 1$

Eq. (5.21) on integration gives

$$(R_2 - R_1) = \left(\frac{m}{32 \pi e^4} \right) \left(\frac{\bar{I}}{m} \right)^2 \frac{A_1 V_o}{Z_1^2 Z_2} \left[Ei(\ln d_1) - Ei(\ln d_2) \right] \qquad \ldots(5.33)$$

where d_1, d_2 or d is given by $d = \left(\frac{2mV^2}{\bar{I}} \right)^2$, Eq.(5.33) is valid for all velocities $V \gg \left(\frac{\bar{I}}{2m} \right)^{1/2}$.

Thus from Eqs. (5.32) and (5.33) the range difference at two different velocities can be obtained, using the exponential integrals.[50]

V.1.4 Range-energy relationship for completely stripped heavy ions

Substituting $z = Z_1$ and the values of the constants in Eqs. (5.32) and (5.33), the range-energy relationship becomes,

(A) For $\chi > 1$

$$\left[R_2 - R_1\right] (mg/cm^2) = (3.494 \times 10^{-5}) \frac{A_1}{Z_1^{2/3}} \frac{A_2 \bar{I}^{4/3}}{Z_2} \left[Ei(\ln C_2) - Ei(\ln C_1)\right] \quad \ldots(5.34)$$

where A_1, A_2 and Z_1, Z_2 are the mass numbers and atomic numbers of the ion and the stopping medium respectively, $\bar{I}$ is the mean ionization potential of the medium in eV and C_1, C_2 or C is given by

$$C = \left[\left(\frac{11.39}{2V_o \bar{I} Z_1}\right)^{2/3} . V^2\right]^2 \quad \ldots(5.35)$$

In Eq. (5.35) V is the velocity of the ion, and $V_o = 2.18 \times 10^8$ cm/sec, both in units of 10^8 cm/sec.

(B) For $\chi < 1$

$$\left[R_2 - R_1\right] (mg/cm^2) = (1.453 \times 10^{-5}) \frac{A_1}{Z_1^2} \frac{A_2 \bar{I}^2}{Z_2} \left[Ei(\ln d_2) - Ei(\ln d_1) \right] \quad \ldots(5.36)$$

where d_1, d_2 or d is given by

$$d = \left[\frac{11.39}{\bar{I}} V^2 \right]^2 \quad \ldots(5.37)$$

V.1.5 Mean ionization potential

Mukherji[51] has derived an expression for the calculation of the mean ionization potentials of elements as follows.

The geometric mean ionization potential $\bar{I}$ of an atom of atomic number Z is given by

$$Z \ln \bar{I} = \sum_s \ln I_s \quad \ldots(5.38)$$

where I_s is the ionization potential of the s^{th} orbital electron of the atom and the summation includes all the Z electrons. Adopting Bohr's[3] procedure, the summation can be replaced by an integration.

$$\sum_s \ln I_s = \int_{all\ U_s} \left\{ \ln \left(\frac{1}{2} m U_s^2 \right) \right\} dn(U_s) \quad \ldots(5.39)$$

where m is the electronic mass, U_s being the velocity of the s^{th} orbital electron. The upper and the lower limits of integration should be the velocities of the innermost and the

outermost electrons in the atom respectively. Since the expressions for $n(U_s)$, given by Eq. (4.16) do not lead to correct values for the velocities of either the innermost or outermost electron, in evaluating Eq. (5.39), it is convenient to consider the contribution of two K-shell electrons separately from that of other (Z-2) electrons.

$$\int_{U_s=0}^{U_s=U_s'} \ln\left(\frac{1}{2} mU_s^2\right) dn(U_s) = \int_{0}^{\frac{(Z-2)V_o}{f(Z)}} \ln\left(\frac{1}{2} mU_s^2\right) dn(U_s) + 2 \ln (13.6) Z^2 \quad \ldots(5.40)$$

The lower limit of integration $U_s=0$ is an approximation made earlier by Bohr.[3] In the case of light elements, the ionization potential I_K of each K-shell electron has been taken as $13.6\ Z^2$eV and this seems to hold in the cases of very heavy elements like uranium, as the electron binding energy data of Hagstrom et al.[52] show. With $dn(U_s) = \frac{f(Z)}{V_o} dU_s$ from Eq.(4.16), Eq. (5.40) gives

$$Z \ln \bar{I} = (Z-2) \ln\left\{\frac{(Z-2)}{2.717\, f(Z)}\right\}^2 .13.6 + 2 \ln 13.6Z^2, \quad ..(5.41)$$

where $\bar{I}$ is in electron volts, Z is atomic number of the atom and f(Z) is given by Eqs. (4.24) and (4.25).

V.2 CALCULATIONS AND COMPARISON WITH EXPERIMENTAL VALUES

Limited number of experimental data[27,28,43] are available on the energy loss measurement of heavy ions in high energy regions, in which the ion is completely stripped. The condition for the ion being completely stripped is given by Eq. (5.23). Table V.1 shows the minimum energy E_m for a few ions at which the ions become completely stripped. The last column of the Table V.1 gives the minimum energy of the ion required for $\chi \simeq 1$. Once the requirement for the ion being completely stripped is fulfilled, the velocity of the ion must also meet either of the conditions $V \gg \sqrt{Z_1 Z_2} V_o$ and $V \gg \frac{Z_1^{1/4} Z_2^{3/4} V_o}{\sqrt{2}}$ for the applicability of Eqs. (5.24) and (5.25). Table V.2 gives the values of the minimum energies E_1 and E_2 required to fulfill the conditions $V \approx \sqrt{Z_1 Z_2} V_o$ and $V \approx \frac{Z_1^{1/2} Z_2^{3/4} V_o}{\sqrt{2}}$ for different incident-ion stopping-medium combinations.

V.2.1 Stopping-powers

Hower and Fairhall[43] have measured the energy degradation of ^{9}Be ions in gold and aluminium foils, Table V.3 shows the experimentally obtained values of the stopping powers and the corresponding calculated values with Eq. (5.26). Experimental stopping powers at different energies were obtained in the following manner. From the incident and emergent energies E_i and E_e of the ion before and after passing through

TABLE V.2

The minimum projectile energies E_1 and E_2, required to fulfill the conditions

$$V \simeq \frac{Z_2 V_o}{2} \chi^{1/3}$$

$$V \simeq \frac{Z_2 V_o}{2} \chi$$

All the energies are in units of MeV/amu.

Ion	STOPPING MEDIUM									
	O_2		Al		Si		Ni		Au	
	E_1	E_2	E_1	E_2	E_1	E_2	E_1	E_2	E_1	E_2
Be	0.56	0.79	1.16	1.29	1.30	1.38	3.67	2.77	17.39	7.8
B	0.62	0.99	1.30	1.61	1.45	1.73	4.10	3.46	19.44	9.78
C	0.68	1.19	1.42	1.93	1.59	2.03	4.49	4.16	21.30	11.74
N	0.74	1.38	1.53	2.25	1.71	2.42	4.85	4.85	23.01	13.70
O	0.79	1.58	1.64	2.57	1.83	2.77	5.19	5.54	24.60	15.65
F	0.84	1.78	1.74	2.89	1.94	3.12	5.50	6.24	26.09	17.61
Ne	0.88	1.98	1.88	3.22	2.05	3.46	5.80	6.93	27.50	19.57

TABLE V.3

Energy (MeV)	(dE/dx) Exp.[43] (MeV cm^2/mg)	(dE/dx) Cal. (MeV cm^2/mg)
9.75	1.13	1.11
13.85	0.90	1.02

a foil of thickness t, the experimental stopping power is obtained as $\frac{(E_i - E_e)}{t}$. The corresponding calculated values have been obtained from Eq. (5.26) using a value of V corresponding to the mean energies $\frac{1}{2}(E_i + E_e)$. Only two experimental data on dE/dx could be obtained in the case of gold, while in the case of aluminium no reliable data could be obtained from the energy vs thickness plot.[43] In a similar manner the energy-degradation data of Northcliffe[27] have been treated to obtain the experimental and calculated stopping powers for the ^{14}N, ^{16}O, ^{19}F and ^{20}Ne ions. Tables V.4 - V.7 show these results in the case of aluminium. The mean ionization potential $\bar{I}$ for aluminium has been calculated with Eq. (5.41) and is 151.4 eV. Kelley et al.[28] have measured the energy loss of ^{12}C, ^{14}N and ^{16}O ions at energies of 2-10 MeV/nucleon in 94.6 μm thick silicon detector. The experimentally obtained values of the stopping powers along with the calculated values, using Eqs. (5.24) and (5.27), are shown in Tables V.8 - V.10. For silicon $\bar{I}$ = 158.8 eV was used in Eqs. (5.24)

TABLE V.4

Experimentally obtained values of the stopping-powers of ^{14}N ion in aluminium along with the calculated values[27]

Energy $(\frac{MeV}{amu})$	(dE/dx) Exp. (MeV cm^2/mg)	(dE/dx) Cal. (MeV cm^2/mg)	Equation used
9.53	1.69	1.76	Eq.(5.27)
8.30	1.81	2.03	Eq.(5.27)
7.46	2.23	2.13	Eq.(5.27)
6.34	2.46	2.42	Eq.(5.27)
4.87	2.95	2.97	Eq.(5.24)
4.12	3.22	3.31	Eq.(5.24)
3.33	3.62	3.76	Eq.(5.24)
2.66	4.42	4.28	Eq.(5.24)
1.85	4.87	5.03	Eq.(5.25)
1.41	5.08	5.11	Eq.(5.26)

TABLE V.5

Experimentally obtained values of the stopping-powers of ^{16}O ion in aluminium along with the calculated values[27]

Energy $(\frac{MeV}{amu})$	(dE/dx) Exp. (MeV cm^2/mg)	(dE/dx) Cal (MeV cm^2/mg)	Equation used
10.25	2.08	2.14	Eq. (5.27)
9.11	2.40	2.38	Eq. (5.27)
8.24	2.64	2.57	Eq. (5.27)
7.47	2.92	2.78	Eq. (5.27)
6.01	3.11	3.28	Eq. (5.24)
4.80	3.73	3.79	Eq. (5.24)
3.65	4.31	4.49	Eq. (5.24)
3.04	4.83	4.99	Eq. (5.24)
2.30	5.77	5.86	Eq. (5.25)

TABLE V.6

Experimentally obtained values of the stopping powers of ^{19}F ion in aluminium along with the calculated values[27]

Energy $(\frac{MeV}{amu})$	(dE/dx) Exp. (MeV cm^2/mg)	(dE/dx) Cal. (MeV cm^2/mg)	Equation used
9.60	2.85	2.89	Eq.(5.27)
9.04	2.98	3.08	Eq.(5.27)
7.92	3.14	3.35	Eq.(5.24)
7.40	3.49	3.51	Eq.(5.24)
6.47	3.82	3.83	Eq.(5.24)
5.49	4.53	4.31	Eq.(5.24)
4.56	4.73	4.80	Eq.(5.24)
3.57	5.94	5.68	Eq.(5.24)
2.48	6.60	6.82	Eq.(5.25)

TABLE V.7

Experimentally obtained values of the stopping-powers of ^{20}Ne ions in aluminium along with the calculated values[27]

Energy $(\frac{MeV}{amu})$	(dE/dx) Exp. (MeV cm^2/mg)	(dE/dx) Cal. (MeV cm^2/mg)	Equation used
9.92	3.51	3.46	Eq.(5.24)
9.23	3.86	3.65	"
8.13	4.00	3.98	"
7.19	4.21	4.32	"
6.20	4.86	5.02	"
5.38	5.04	5.22	"
3.50	6.51	6.74	"

TABLE V.8

Experimentally obtained values of the stopping-powers of ^{12}C ion in 94.6 μm thick Si-detector along with the calculated values using Eq. (5.27)[28]

E_i (MeV)	ΔE (MeV)	$\bar{E}=E_i-\frac{\Delta E}{2}$ (MeV)	(dE/dx) Exp. (KeV cm^2/mg)	(dE/dx) Cal. (KeV cm^2/mg)
120.0	32.0	104.0	1451	1432
109.2	35.0	91.7	1588	1580
102.0	36.0	84.0	1633	1691
92.0	38.0	73.0	1724	1888
84.0	49.0	60.5	2223	2179
78.0	54.0	51.0	2449	2482

TABLE V.9

Experimentally obtained values of the stopping-powers of ^{14}N ion in 94.6 μm thick Si-detector along with the calculated values[28]

E_i (MeV)	ΔE (MeV)	$\bar{E}=E_i-\frac{\Delta E}{2}$ (MeV)	(dE/dx) Exp. (KeV cm^2/mg)	(dE/dx) Cal. (KeV cm^2/mg)
140.0	42.5	118.8	1928	1999
127.0	48.0	103.0	2178	2237
120.0	50.0	95.0	2276	2382
110.0	56.0	82.0	2541	2673
100.0	65.0	67.5	2951	3099

TABLE V.10

Experimentally obtained values of the stopping-powers of ^{16}O ion in 94.6 μm thick Si-detector along with the calculated values[28]

E_i (MeV)	ΔE (MeV)	$\bar{E}=E_i-\frac{\Delta E}{2}$ (MeV)	(dE/dx) Exp. (KeV cm^2/mg)	(dE/dx) Cal. (KeV cm^2/mg)	Equation used
155.0	56.0	126.0	2630	2736	Eq.(5.27)
140.0	68.0	106.0	3083	3221	Eq.(5.27)
132.0	77.0	94.5	3491	3436	Eq.(5.24)
122.0	86.0	79.0	3899	3856	Eq.(5.24)
110.0	100.0	60.0	4539	4564	Eq.(5.24)

and (5.27), for the calculation of stopping powers. The value of $\bar{I}$ was obtained using Eq. (5.41).

V.2.2 Range-differences

Table V.11 shows the range difference (R_2-R_1) at two different energies (E_2-E_1) for ^{10}B, ^{11}B, ^{14}N, ^{16}O, ^{19}F, and ^{20}Ne ions in aluminium. The experimental values are obtained from the energy-loss data of Northcliffe ,[27] while the calculated values are obtained using either Eq. (5.34) or Eq.(3.36) depending on the values of χ greater or less than unity. The exponential integrals were evaluated using the "Tables of sine, cosine and exponential integrals".[50] The uncertainty in the experimental ranges are not shown in the table.

V.3 DISCUSSION

The stopping power equations given by Eqs. (5.24), (5.25) and (5.26) can be used for any completely stripped ion with $\chi > 1$, while Eq. (5.27) is valid only for those ions for which $\chi \ll 1$. The condition for $\chi < 1$ for ions heavier than protons and alpha particles always fulfill the condition given by Eq. (5.23), i.e., for the ion to be in a completely stripped state. The agreement between the calculated and the experimentally obtained values of the stopping powers is usually good.

TABLE V.11

Range-Differences for heavy ions in aluminium in terms of $(R_2)-(R_1)^{27}$.

Ion	Energy interval (MeV/amu)	Range-Exp. (mg/cm^2)	Range-Cal. (Mg/cm^2)	Equation used
^{10}B	9.73-2.92	56.97	54.36	Eq.(5.36)
^{11}B	9.88-2.48	66.49	64.03	Eq.(5.36)
^{11}B	2.48-1.68	3.53	3.21	Eq.(5.34)
^{14}N	9.00-4.74	36.72	35.82	Eq.(5.36)
^{14}N	4.74-1.65	48.70	46.61	Eq.(5.34)
^{16}O	9.41-6.34	25.06	24.15	Eq.(5.36)
^{16}O	6.34-2.55	41.49	39.21	Eq.(5.34)
^{19}F	9.69-8.04	14.14	13.51	Eq.(5.36)
^{19}F	8.04-3.00	37.92	35.98	Eq.(5.34)
^{20}Ne	9.35-3.17	31.89	30.52	Eq.(5.34)

CHAPTER VI

EFFECTIVE CHARGE

The charge of a fast ion moving through matter fluctuates as a result of the competition between electron loss and electron capture in collisions with the atoms of the stripping target. After the ions have made a sufficient number of collisions with the target atoms an equilibrium distribution of charges is established which is dependent only on the velocity of the ions. This equilibrium distribution of charges does not change when the target thickness is further increased. The mean equilibrium charge $\bar{q}$ (equilibrium charge) is obtained from the equilibrium charge distributions. For the purpose of stopping power calculations, $\bar{q}$ is usually identified with the "effective charge" (Z^{eff}) of the ion which has been described earlier.

The excellent agreement of the calculated values of the stopping powers of various ions (^{9}Be to ^{238}U) with the corresponding experimental values in different solid media except

carbon and beryllium, seems to provide very strong, although indirect, evidence in favour of the assumption that

$$Z^{eff} = f(Z) \frac{V}{V_o} \qquad \ldots(6.1)$$

where f(Z) is given by Eqs. (4.24) and (4.25). The fact that the light gases, as shown in Fig. 1, also obey Eq. (4.20), possibly indicates that Eq. (6.1) is independent of the physical states of the medium. In the following section a comparison of the measured equilibrium charges $\bar{q}$ and the corresponding values of Z^{eff} calculated from Eq. (6.1) has been made to establish the validity of Eq. (6.1).

VI.1 Comparison of the directly measured equilibrium charges and the calculated effective charges

Tables VI.1 - VI.6 show some experimental values of $\bar{q}_g$ and $\bar{q}_s$ from some authors[53-57] along with our calculated values using Eq. (4.22), where $\bar{q}_g$ and $\bar{q}_s$ are the equilibrium charges of an ion after it emerges from a gaseous and a solid medium. In some cases the standard deviations σ associated with the measured equilibrium charges are also listed in the tables. The experimental set up for the measurement of $\bar{q}$ consists of the following:

(a) A magnetic analyzer to select accelerated ions of unique charge states.

(b) A stripper chamber which contained a foil and a gas cell.

(c) An analyzer to separate the beam emerging from the foil or the gas stripper into the various charge states.

(d) A detection system to determine the relative intensi ties of the charge state components. In this way post stripper charge distributions and their corresponding pre-stripper charg can be obtained for different energies simultaneously.

From the Tables VI.1 - VI.6 it is evident that for all ions the agreement between $\bar{q}_g$ and the calculated Z^{eff} is good but Z^{eff} differs considerably from $\bar{q}_s$. This difference in the values of Z^{eff} and $\bar{q}_s$ can be explained if it is assumed that <u>inside</u> a solid medium the equilibrium charge does not differ appreciably from the equilibrium charge in a gaseous medium. Betz,[58] in his review work, has agreed with the conclusion of Betz and Grodzins[59] that inside a solid medium the equilibrium charge is possibly the same as that in a gaseous medium. The gas-solid difference has been attributed to the fact that a suff cient time elapses between inelastic collisions in a gas for ion to de-excite to their ground states whereas in a solid many coll sions take place while the ions are in the excited state from which electrons are readily stripped. The higher value of the equilibrium charge when the ion emerges from a solid stripper ma be attributed to the surface effects.[60]

TABLE VI.1

Directly measured equilibrium charges ($\bar{q}_g$) in gaseous media for various ions along with the calculated values of the effective charge Z^{eff} at different energies

Ions	Velocity x 10^{-8} cm/sec.	He		N_2		Ar		Kr		Z^{eff} (Calcd)
		Z^{eff}	σ	Z^{eff}	σ	Z^{eff}	σ	Z^{eff}	σ	
^{7}Li [a]	4.01	1.12	0.46	1.25	0.50	1.27	0.49	1.14	0.44	1.07
	5.69	1.52	0.58	1.73	0.57	1.82	0.56	1.66	0.56	1.52
	8.03	2.17	0.62	-	-	-	-	2.28	0.57	2.14
	8.06	-	-	2.38	0.59	2.50	0.56	-	-	2.15
	11.64	2.81	0.41	2.80	0.42	2.77	0.44	2.64	0.52	3.11
^{11}B [a]	2.75	1.13	0.61	1.23	0.66	1.20	0.60	1.05	0.58	1.03
	3.79	1.35	0.67	1.56	0.68	1.57	0.67	1.45	0.65	1.42
	7.00	2.55	0.67	-	-	2.85	0.60	2.80	0.58	2.63
	7.87	2.85	0.65	3.03	0.62	3.10	0.59	3.02	0.57	2.95
^{14}N [a]	2.60	1.2	-	1.3	-	1.3	-	1.0	-	1.22
	4.11	1.6	-	2.0	-	1.9	-	1.6	-	1.93
	5.69	2.4	-	2.7	-	2.8	-	2.6	-	2.67
	7.02	2.9	-	3.5	-	3.6	-	3.6	-	3.29
	7.91	3.5	-	3.8	-	4.2	-	3.8	-	3.72
	9.15	4.0	-	4.4	-	4.5	-	4.4	-	4.30
	11.64	4.8	-	4.9	-	4.9	-	4.6	-	5.47

a, Nikolaev et al., JETP, 12, 627 (1961).

TABLE VI.2

Directly measured equilibrium charges ($\bar{q}_g$) in gaseous media for various ions along with the calculated values of the effec--tive charge Z^{eff} at different energies

Ions	Velocity x 10^{-8} cm/sec	He	N_2	Ar	Kr	Z^{eff} (Calcd)
^{16}O [a]	6.29	-	-	3.5±0.766	-	3.23
	8.55	-	-	4.8±0.766	-	4.39
	10.21	-	-	5.2±0.766	-	5.24
^{20}Ne	2.65 [b]	1.6	1.7	1.7	1.5	1.58
	4.04 [b]	2.2	2.2	2.5	2.2	2.41
	5.45 [a]	-	-	3.3±0.766	-	3.25
	5.52 [b]	2.7	3.5	3.5	3.3	3.29
	6.33 [b]	3.4	4.0	- -	-	3.77
	8.25 [a]	-	-	4.9±0.766	-	4.92
	9.94 [a]	-	-	6.0±0.766	-	5.93

a, Hubbard and Lauer, Phys. Rev., 98, 1814 (1955);
b, Nikolaev et al., JETP 12, 627 (1961).

TABLE VI.3

Directly measured equilibrium charges of ^{23}Na, ^{31}P and ^{40}Ar ions in N_2 ($\bar{q}_g$) along with the calculated values of Z^{eff}.

Ions	Velocity x 10^{-8} cm/sec	$\bar{q}_g$	Z^{eff}(Calcd)
^{23}Na [a]	4.5	3.1	2.86
^{31}P [a]	4.05	3.0	3.16
^{40}Ar [a]	4.05	3.2	3.57

a, Nikolaev et al., JETP, 12, 627 (1961).

TABLE VI.4

Directly measured equilibrium charges of ^{27}Al ions in $N_2(\bar{q}_g)$ along with the calculated values of Z^{eff} at different energies

Energy (MeV)	0.80	1.14	1.40	1.72	2.09	2.35	2.65	3.03	3.35	3.72	4.00
$\bar{q}_g$ [a]	1.80	2.06	2.29	2.52	2.78	2.91	3.08	3.32	3.50	3.67	3.85
σ	0.72	0.75	0.76	0.78	0.81	0.82	0.83	0.87	0.87	0.90	0.91
Z^{eff} (Calcd)	1.69	2.02	2.24	2.48	2.74	2.90	3.08	3.29	3.46	3.65	3.78

a, G. Ryding et al., Phys. Rev. A 1, 1081 (1971).

TABLE VI.5

Directly measured[a] equilibrium charges of ^{32}S ions in air ($\bar{q}_g$) and carbon ($\bar{q}_s$) along with the calculated values of Z^{eff} at different energies

Energy (MeV)	$\bar{q}_g$(air)	$\bar{q}_s$(carbon)	Z^{eff}(calculated)
3.70	3.45	5.85	3.84
6.40	4.65	6.85	5.05
8.40	5.16	7.45	5.78
10.00	5.60	7.80	6.31
12.70	6.50	8.45	7.11
17.00	7.50	9.09	8.23
20.00	8.06	9.45	8.92
22.70	8.50	9.71	9.51
23.80	8.65	9.87	9.73
26.40	8.90	10.14	10.26
27.20	8.80	10.25	10.41
29.90	9.28	10.46	10.91
32.60	9.58	10.66	11.40

a, Betz et al., Phys. Letters, 22, 643 (1966).

TABLE VI.6

Directly measured[a] equilibrium charges of ^{35}Cl ion in gases ($\bar{q}_g$) and carbon ($\bar{q}_s$) along with the calculated values of Z^{eff} at different energies

Energy (MeV)	H_2		He		N_2		O_2		Ar		Kr		C		Z^{eff} (Calcd)
	$\bar{q}_g$	σ	$\bar{q}_g$	σ	$\bar{q}_g$	σ	$\bar{q}_g$	σ	$\bar{q}_g$	σ	$\bar{q}_g$	σ	$\bar{q}_s$	σ	
2.00	2.15	0.893	2.36	0.956	2.76	1.03	2.78	1.04	2.68	1.02	2.33	0.954	4.61	1.38	2.81
4.00	3.26	0.890	3.13	0.958	3.86	1.08	3.74	1.06	3.87	1.05	3.54	1.01	6.02	1.26	3.97
6.00	4.20	0.888	3.87	0.983	4.60	1.05	4.46	1.06	4.76	1.04	4.46	1.02	6.69	1.21	4.87
8.00	5.08	0.876	4.59	1.00	5.34	1.06	5.18	1.10	5.54	1.03	5.56	1.03	7.56	1.16	5.62
10.00	5.79	0.855	5.21	1.00	5.77	1.07	5.66	1.09	6.01	1.02	6.16	0.942	8.09	1.17	6.28
12.00	6.41	0.858	5.78	0.994	6.54	1.07	6.33	1.10	6.77	0.995	6.80	1.02	8.53	1.17	6.88

a, Wittkower and Ryding, Phys. Rev., A4, 226 (1971).

TABLE VI.7

Directly measured[a] equilibrium charges of ^{79}Br ion in gases ($\bar{q}_g$) and carbon ($\bar{q}_s$) along with the calculated values of Z^{eff} at different energies.

Energy (MeV)	H_2		He		N_2		O_2		Ar		Kr		C		Z^{eff} (Calc
	$\bar{q}_g$	σ	$\bar{q}_g$	σ	$\bar{q}_g$	σ	$\bar{q}_g$	σ	$\bar{q}_g$	σ	$\bar{q}_g$	σ	$\bar{q}_g$	σ	
2.00	1.46	0.83	2.25	0.99	2.20	1.18	2.42	1.23	1.96	1.04	1.60	0.95	5.74	1.63	3.0
4.00	2.01	0.91	2.64	1.07	3.10	1.39	3.27	1.44	2.97	1.38	2.45	1.28	7.73	1.50	4.2
6.00	2.71	0.97	3.17	1.12	4.08	1.58	4.06	1.58	3.82	1.57	3.42	1.52	8.95	1.43	5.2
8.00	3.29	0.97	3.75	1.13	4.85	1.63	4.80	1.68	4.70	1.66	4.23	1.68	9.69	1.43	6.0
10.00	3.95	0.97	4.32	1.17	5.44	1.62	5.45	1.63	5.43	1.72	5.04	1.76	10.40	1.47	6.7
12.00	4.70	0.98	4.83	1.20	6.03	1.62	6.09	1.66	6.08	1.70	5.94	1.77	11.20	1.49	7.4
14.00	5.29	0.99	5.38	1.17	6.44	1.57	6.66	1.72	6.66	1.72	6.55	1.73	-	-	8.0

a, Wittkower and Ryding, Phys. Rev., A4, 226 (1971).

VI.2 Discussion

According to many authors[8,48,49,60,61] the stopping power of a heavy ion is related to the stopping power of proton at the same velocity and in the same stopping medium by

$$\frac{(Z^{eff})^2_{HI}}{(Z^{eff})^2_{p}} = \frac{(dE/dx)_{HI}}{(dE/dx)_{p}} , \qquad \ldots(6.2)$$

where the subscripts HI and p refer to the heavy ion and the proton respectively. Eq. (6.2) is obtained from Bethe's equation, in which the logarithmic part is independent of the charge of the ion. Eq. (6.2) has been utilized to make an estimate of Z^{eff} from the measured stopping powers and for obtaining the energy-loss and ranges of heavy ions through interpolations and extrapolations. Bohr[3] pointed out that Eq. (3.12) is applicable if $\chi \ll 1$ or $2f(Z_1) \ll 1$, a condition which is not always fulfilled in the case of partially stripped heavy ions. On the other hand, if $2f(Z_1) \gg 1$, which is marginally valid for most heavy ions, Eq. (4.37), which is derived from Eq. (4.32), can be utilized for stopping power calculations. Since Eq. (4.37) predicts stopping powers that are in fair agreement with the experimental values, a comparison between Eqs. (4.32) and (3.12) shows that the assumption of the logarithmic term being simply velocity dependent may not be strictly valid. The values of Z^{eff} obtained with Eq.(6.2) may, therefore be somewhat different from the actual values inside a solid medium.

CHAPTER VII

S U M M A R Y

Using Bohr's semi-quantum mechanical expression for the energy loss of a swiftly moving charged particle a set of stopping power and range-velocity equations have been deduced. These equations predict stopping powers with an average deviation of less than 5% from the experimental values. The range-velocity and stopping power equations are of great practical importance in a number of fields, such as in heavy ion acceleration, in developing fission electric cells for power generation and in the field of radiation bilogy, since these equations allow quantitative prediction of the amount of energy deposited by fast moving ions and fission fragments in any compound media of known molecular composition like proteins. Heavy ion beams are potentially the most suitable form of "radiation" for delivering a highly localized dose to any deep seated region of choice in the body. The following are the stopping power and range-velocity

equations:

Completely stripped heavy ions

(A) $V < 2Z_1V_o$

(a) $V > \sqrt{Z_1Z_2}V_o$

$$\frac{dE}{dx} = 63.65 \frac{Z_1^2 Z_2}{A_2 V^2} \log \left(\frac{11.39V^3}{2Z_1V_o\bar{I}}\right)$$

$$\left[R_2-R_1\right] = (3.494\times10^{-4}) \frac{A_1}{Z_1^{2/3}} \frac{A_2\bar{I}^{4/3}}{Z_2} \left[Ei(\ln C_2)-Ei(\ln C_1)\right]$$

where C_1, C_2 or in general C is given by

$$C = \left[\left(\frac{11.39}{2V_o\bar{I}Z_1}\right)^{2/3} V^2\right]^2$$

(b) $\sqrt{Z_1Z_2}V_o \gg V \gg \frac{Z_1^{1/4} Z_2^{3/4} V_o}{\sqrt{2}}$

$$\frac{dE}{dx} = 13.79 \frac{Z_1^2}{A_2V^2} \left[3(Z_2-2) + \frac{f(Z_2)}{Z_1} \left(\frac{V_o}{V}\right)^2 + 2.303 \left\{6 \log \frac{2V}{Z_2V_o} + 3(Z_2-2) \log \frac{2Vf(Z_2)}{V_o(Z_2-2)} - Z_2 \log 2Z_1 \frac{V_o}{V}\right\}\right]$$

(c) $V \ll \frac{Z_1^{1/4} Z_2^{3/4} V_o}{\sqrt{2}}$

$$\frac{dE}{dx} = 12.68 \frac{f(Z_2)}{A_2} \frac{Z_1^2}{V} \left[3\left\{2 Z_1 \frac{V_o}{V}\right\}^{-1/3} + \left\{2 Z_1 \frac{V_o}{V}\right\}^{-1}\right]$$

(B) $V > 2Z_1V_o$

$$\frac{dE}{dx} = 63.65 \frac{Z_1^2 Z_2}{A_2 V^2} \log \left(\frac{11.39\ V^2}{\bar{I}} \right)$$

$$\left[R_2 - R_1 \right] = (1.435 \times 10^{-5}) \frac{A_1}{Z_1^2} \frac{A_2 \bar{I}^2}{Z_2} \left[Ei(\ln d_2) - Ei(\ln d_1) \right]$$

where d_1, d_2 or in general d is given by

$$d = \left(\frac{11.39}{\bar{I}} V^2 \right)^2$$

and holding for $V \gg \left(\frac{\bar{I}}{11.39} \right)^{1/2}$

Partially stripped ions and fission fragments

$$\frac{dE}{dx} = 1.327 \frac{f(Z_2)}{A_2} \left[4.7622 \{ f(Z_1) \}^{5/3} + f(Z_1) \right] V$$

$$R = \frac{A_1 A_2 (V_i - V_o)}{127.3\ f(Z_2) \left[4.7622 \{ f(Z_1) \}^{5/3} + f(Z_1) \right]}$$

In all these equations $\frac{dE}{dx}$ is in units of $\frac{\text{MeV cm}^2}{\text{mg}}$, while $(R_2 - R_1)$ or R is in units of mg/cm^2. A_1, A_2 and Z_1, Z_2 are the mass numbers and the atomic numbers of the ion and the medium respectively. V_i is the initial velocity of the ion, V is the velocity at which the stopping power is being calculated, and $V_o = e^2/\hbar(2.18 \times 10^8 \text{ cm/sec})$. V_i, V and V_o are to be expressed in units of 10^8 cm/sec. $\bar{I}$ is the mean ionization potential of

the medium in eV and $f(Z_2)$ or $f(Z_1)$ or in general $f(Z)$ is given by

$$f(Z) = 0.28\, Z^{2/3} \quad \text{for } Z \ll 45.5$$

$$f(Z) = Z^{1/3} \quad \text{for } Z \gg 45.5$$

The equations given above are applicable in the case of compound media if A_2 and $f(Z_2)$ are replaced by $\Sigma_i X_i A_i$ and $\Sigma_i X_i f(Z_i)$ respectively, where the compound is made up of X_i atoms of the i^{th} atomic species having mass number and atomic number A_i and Z_i, respectively.

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